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A Simple Economic Theory of Skill Accumulation and Schooling Decisions[1]

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Abstract

We propose a model of schooling that can account for the observed heterogeneity in workers' productivity and educational attainment. Identical unskilled agents can get a degree at a cost, but becoming skilled entails an additional unobservable effort cost. Individual labor can then be used as an input in pairwise production matches. Two factors affect students' desire to build human capital: degrees imperfectly signal productivity, and contract imperfections generate holdup problems. Multiple stationary equilibria exist, some of which are market failures characterized by a largely educated workforce of low average skill. Policy implications are explored.

Keywords: Education Policy, Education Finance, Human Capital, Informational Frictions, Matching, Multiple Equilibria.

JEL: D8, I2, J24

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"You've gone to the finest school all right...but you know you only used to get juiced in it."
−Bob Dylan (Like a Rolling Stone).

1 Introduction

This paper offers a new perspective on factors that can account for the observed heterogeneity in workers' productivity and educational attainment. It develops a theoretical study of a matching economy in which education has both a productive value (Becker, 1964), and an identification value (Arrow, 1973, Spence, 1973, Stiglitz, 1975). The objective is to examine the schooling and human capital decisions that may result when education facilitates the buildup of imperfectly recognizable skills by means of unobservable effort.

In our model, workers can acquire an education—and so obtain a degree—at some cost. However, becoming skilled entails an additional unobservable effort cost. Thus, student achievement is an economic decision that is complementary to the schooling choice, but which is imperfectly reflected in the possession of a degree. The central result is that even if we start with a homogenous population the economy can end up with a largely educated workforce of low average skill. The economic mechanism is intuitive. First, since student effort is unobservable, a degree gives a potential employer only vague information on the worker's productivity. Second, if contractual imperfections exist, these create holdup problems that let the less efficient workers capture some productivity rents. These two frictions weaken the incentives to academic achievement, and can account for market failures characterized by disparities in educational attainment and skills.

Prior research has identified factors capable of generating market failures in the acquisition and provision of productive skills.[2] That schooling and skill accumulation are not necessarily one and the same, is also a long-standing notion—which is perhaps why Mark Twain remarked "I have never let my schooling interfere with my education." Our contribution is to embed this notion into a rigorous theoretical framework that builds intuition on an important theme of the U.S. education debate: how informational problems may weaken the incentives to academic achievement and

[2] In sorting models of education differences in innate abilities and informational asymmetries may generate a wedge between private and social returns to skills (Weiss, 1995). Firms' imperfect competition for labor, individuals' credit constraints, and matching externalities are also factors that may lead to market failures, as in Booth and Snower (1995) or Fernandez and Gali (1999). In the latter, especially, borrowing constraints generate market failures in allocating ex-ante heterogeneous students to schools of different quality.

inhibit human capital buildup (see for example Owen, 1995). Indeed, in our model the equilibrium values of a degree and of skill—to any worker—depend on the unobservable effort choices of the student population.

There are, of course, many studies on the links between the educational process, disparities in human capital, and attainment (see the survey of Weiss, 1995). Broadly speaking, these fall into one of two classes of models. In one, diversity in schooling choices simply reflects innate skill disparities. In the other, education can facilitate human capital buildup, but again disparities in productivity or schooling largely reflect exogenous factors.[3] As in our paper, human capital accumulation in such models suffers when incentives to student achievement are weak. For example see the studies by Betts (1998) and Sahin (2003) on the economic impact of education standards and financing. What sets our model apart is the root of productivity disparities. Rather than exogenous differences or imperfections in the education technology, the driving force is the agents' desire to earn productivity rents by mimicking the more productive workers (earning a degree) while minimizing study effort (low achievement). If contractual imperfections allow too frequent a redistribution of surplus from the more to the less productive workers, then accomplishing little while in school is individually optimal, but socially suboptimal.

The analysis generates suggestions for education policy. The first consideration is familiar to the U.S. debate (e.g. see the Commission on the Skills of the American Workforce, 1990): the educational system should strive to provide incentives to student achievement. Second, the model suggests the importance of policies directed at diminishing informational asymmetries, for example by raising education standards or the informativeness of academic certificates. We also find that increased public effort to lower the private cost of education may be ineffective in raising the workforce's skill level, unless accompanied by complementary incentives to student performance.

2 A Snapshot of the Model

We consider a large population of ex-ante unskilled workers. Agents can choose to acquire an education—and thus obtain a degree—at some cost, but becoming skilled entails an additional

[3] For example, there can be payoff-irrelevant factors, as the observable immaterial features (e.g. color of skin) of Moro and Norman's (2003) statistical discrimination model, or payoff-relevant factors as in Blankenau (1999). There can be also factors intrinsic to the skill acquisition process, as the random factors in Lazear and Rosen (1981). Finally, heterogeneity can depend on a mixture of the elements mentioned above, as in Weiss (1983).

unobservable effort cost. Thus, degrees are imperfect signals of productivity. To enhance earnings over autarky, workers market their labor by means of a random process that pairs everyone to someone different at every date. When two workers meet they simultaneously propose whether to form a production partnership, interpreted as a firm. The alternative is autarkic production.

In a production partnership, skilled labor is necessary to generate surplus. Imperfections in the contracting process, however, create a holdup problem in 'mismatched' partnerships: the less productive agent captures some of her partner's ability rents (e.g. see Camera et al. 2003). Thus, the acquisition of skill generates a positive externality to any partner, and everyone tries to team up with skilled workers. Since productivity is imperfectly observable, however, not all students might choose to augment their productivity. This causes adverse selection since the unskilled do not sort themselves out of the market.

We study all the stationary equilibria, and find that equilibrium skill heterogeneity is linked to the use of mixed strategies. The contractual imperfections and the degrees' ineffectiveness in attesting skill create an incentive to free ride by mimicking productive workers, going to school without building skill. Realizing this, the market forms expectations on the probability that educated workers are skilled. Lower costs of education or greater market imperfections make it easier to earn undeserved productivity rents on the market, and so raise the incentive to free ride.

An additional reason to under-invest in skill works its way though market expectations. Indeed, fears of extensive free-riding behavior can be self-fulfilling since human capital accumulation is less attractive when workers expect a great incidence of holdup problems. In particular, since the equilibrium return to skill depends on the endogenous productivity disparities, and since private choices are uncoordinated, there are strategic complementarities in education decisions. This generally leads to multiple equilibria, some of which are market failures with an heterogeneously productive educated workforce. The lower is average productivity, the greater the inefficiency.

3 Environment

Time is discrete and continues forever. There is a constant population comprised of a large even number of ex-ante identical *unskilled* agents. They produce a homogeneous perishable and divisible good, have identical preferences that are linear in consumption, and discount the future at rate $\beta \in (0, 1)$. Each agent faces a constant probability $1 - \pi$ of exiting the economy at the end of each period, being replaced by a newborn unskilled agent. Thus, agents spend an average

of $(1-\pi)^{-1}$ periods in the economy and leave it in finite time with probability one.

At the beginning of life each agent can choose to permanently enhance her productivity by undertaking a costly educational opportunity. She can acquire a *degree* and, contingent on that, can invest additional resources to earn productivity-enhancing *skills*. Earning a degree—via a process we call *schooling*—generates disutility $c_d > 0$ but does not increase productivity *per se*. To acquire skill the agent must suffer additional disutility $c_s > 0$. These decisions are made simultaneously, and the outcome is instantaneous so that—after her initial choice—the agent can remain unskilled (with low-productivity) or can become skilled (with high-productivity).[4] Consequently, agents can be divided into three types; type $i = n$ if the agent has no degree, $i = d$ if she has a degree but no skills, and $i = s$ if she is skilled and has a degree.

Following the initial education/skill choice agents enter a market where they can promote their productive abilities. We assume that at each date every agent is paired to an anonymous partner according to an exogenous matching process as specified in Aliprantis et al. (2004). Degrees are observable, individual histories are not, and skills are observable with probability $\gamma \in [0,1]$, independent across agents and matches.[5] Thus one, both, or no agent may be informed about the partner's productivity, and no one can directly observe whether the partner is informed.

When two individuals meet, they play a coordination game. Each agent independently and simultaneously proposes one of two mutually exclusive productive activities: (i) costless *autarkic* production or (ii) setting up jointly a temporary firm to engage in costless and instantaneous *market* production. If the proposals match they are implemented, else both agents produce in autarky. Also, opting for market production precludes the possibility of reverting to autarky during the period.

To introduce the payoffs from the different productive activities, let u_i be the utility associated

[4] A natural interpretation of c_d is the value of forgone unskilled wages and tuition payments, while c_s captures the existence of a quantifiable level of disutility from supplying effort while in school. Introducing innate productivity differentials, or letting skilled agents further augment their productivity, would increase the dimensionality of the state space, complicating the exposition but providing little additional insight. This is also why we let the process of education be instantaneous (as is done, for example, in Lazear and Rosen, 1981, or Costrell, 1994).

[5] This is a standard way to capture the idea of a lemons' problem in bilateral matches (e.g. Williamson and Wright, 1995). Agents are sometimes unable to recognize the 'quality' of their partner's productivity. It may be taken to capture the efficiency of a publicly observable testing procedure used to ascertain the skill of those schooled.

with autarkic consumption by an agent in state i (agent i for short). Since only skills can enhance productivity, it is assumed that $u_s > u_d = u_n \geq 0$. Let $G_i(k) = g_i(k) - u_i$ be agent's i surplus when she is in a match with agent $k \in \{n, d, s\}$. It is the difference between i's utility from eating her share of the firm's output, $g_i(k)$, and autarkic utility. We assume complementarities in joint production with skills, and increasing returns in the firm's skill level. The key implication is that *only* skills can generate surplus in a match, and it is the highest in matches between two skilled agents.

We model contracting imperfections, by postulating that skilled agents lose some of their ability rents to less productive partners. A straightforward way to implement this is to simply assume values of $g_i(k)$ and u_i that support the following:

Match\Agent	n	d	s
Cross	$G_n(s) > 0$	$G_d(s) > 0$	$G_s(d) < 0$
Self	$G_n(n) = 0$	$G_d(d) = 0$	$G_s(s) > 0$

where *cross-matches* have agents of different productivity (unlike *self-matches*) and

$$2G_s(s) > \max(0, G_d(s) + G_s(d)). \tag{1}$$

That is, while cross-matches might (or might not) generate surplus, they generate less surplus than skilled self-matches. Either way, the more productive agent always suffers a loss while the less productive worker always realizes a gain.[6]

4 Symmetric Stationary Equilibria

We study equilibria where individuals adopt time-invariant symmetric strategies taking market payoffs and strategies of others as given. Individual actions are based on the correct evaluation of the gains associated with each possible match, strategies of others and distributions.

Denote the stationary educational choices by $\delta' \in [0, 1]$, the probability that the representative agent acquires a degree at the beginning of life, and $\sigma' \in [0, 1]$, the conditional probability that the agent also invests in skill. Let $\omega' \in [0, 1]$ be the probability that a skilled agent proposes to set up a firm with someone whose skills are unrecognized. We let (δ, σ, ω) denote the vector of

[6] For example, we may postulate that agents split equally the firm's output, in which case $g_s(n) = g_n(s)$ and $g_d(n) = g_n(d)$, so that (1) is implied by $2g_s(s) > 2u_s > 2g_s(n)$ and $u_s + u_n > 2g_d(n) = 2g_n(n) = 2u_n$.

probabilities chosen by everybody else in the market, where we use the superscript '*' to indicate a variable taking values in the open unit interval, i.e. $\delta^* \in (0,1)$ etc. The population is assumed sufficiently large that, by Kolmogorov's law of large numbers, we can treat the probabilities as population proportions; that is, δ is the proportion of the educated population, while $P_s = \delta\sigma$ and $P_d = \delta(1-\sigma)$ are the population proportions that have skill and only a degree. Finally, let V_i denote the expected stationary lifetime utility of an agent in state $i = n, d, s$.

4.1 Individually Optimal Strategies

Consider a representative agent. Because the choice of education and skill are intertwined, it is convenient to break up the problem into two parts. Contingent on investing in the educational opportunity, the agent must choose whether to invest in skill. Given the strategies of all others (δ, σ, ω), she acquires skill if it improves her expected lifetime utility, over that associated to mere ownership of a degree. Her best response correspondence is:

$$\sigma' = \begin{cases} 1 & \text{if } V_s - c_s > V_d \\ [0,1] & \text{if } V_s - c_s = V_d \\ 0 & \text{if } V_s - c_s < V_d. \end{cases} \tag{2}$$

Similar reasoning implies her optimal choice of schooling must satisfy:

$$\delta' = \begin{cases} 1 & \text{if } V_n < \max\{V_d, V_s - c_s\} - c_d \\ [0,1] & \text{if } V_n = \max\{V_d, V_s - c_s\} - c_d \\ 0 & \text{if } V_n > \max\{V_d, V_s - c_s\} - c_d. \end{cases} \tag{3}$$

Now, recall that those unskilled earn surplus only in cross-matches. It follows that a less productive agent always proposes to team up with someone who has or might have skills, while we assume she chooses autarky otherwise (a small transaction cost would endogenize this).

Since $G_s(s) > 0$, someone skilled strictly prefers to form a production partnership when she is aware of being in a self-match. However, because $G_s(d) < 0$ she does not *knowingly* participate in cross-matches. When her partner's skills are unobserved she proposes market production if her interim participation constraint—non-negativity of expected surplus—is satisfied. Her optimal

choice must satisfy:

$$\omega' = \begin{cases} 1 & \text{if } \; P_s\left[\gamma + (1-\gamma)\,\omega\right] G_s(s) + P_d G_s(d) > 0 \\ [0,1] & \text{if } \; P_s\left[\gamma + (1-\gamma)\,\omega\right] G_s(s) + P_d G_s(d) = 0 \\ 0 & \text{if } \; P_s\left[\gamma + (1-\gamma)\,\omega\right] G_s(s) + P_d G_s(d) < 0. \end{cases} \tag{4}$$

The expected market surplus from teaming up with an educated agent of unobserved skill has two elements. The first, $P_s[\gamma + (1-\gamma)\omega]G_s(s)$, is positive. It is the expected gain from getting a good job, i.e. joining in a productive firm. The second, $P_d G_s(d)$, is negative and reflects the losses she will suffer if, with probability P_d, she joins a less productive partnership. Symmetry requires

$$\left(\sigma', \delta', \omega'\right) = (\sigma, \delta, \omega)\,. \tag{5}$$

4.2 Value Functions

Following the discussion above, and letting $r = 1 - \beta\pi$, standard dynamic programming techniques imply that the value functions must satisfy:

$$\begin{aligned} rV_s &= u_s + \gamma P_s[\gamma + (1-\gamma)\omega]G_s(s) + (1-\gamma)\max_{\omega'} \omega' \left\{P_s[\gamma + (1-\gamma)\omega]G_s(s) + P_d G_s(d)\right\} \\ rV_d &= u_n + (1-\gamma)P_s\omega G_d(s) \\ rV_n &= u_n. \end{aligned} \tag{6}$$

The first line indicates the lifetime flow return to a skilled worker hinges on three elements. The first is positive and deterministic, as she can always produce in autarky netting period utility u_s. The remaining are stochastic components, associated with the gains expected from marketing skills. The second term is the surplus expected from setting up a firm with partners who are known to be equally skilled. The third term is non-negative, and it captures the gain expected from matching with educated workers of unknown productivity. The other lines of (6) are interpreted similarly.

Comparing V_d and V_n, note that someone stands to gain from earning *solely* a degree, only if (i) there are skills in the economy that can sometimes go undetected, $P_s > 0$ and $\gamma < 1$, and if (ii) cross-matches can be formed, $\omega > 0$. Since $G_d\,(s) > 0$, it may be worthwhile to undertake the educational opportunity without exploiting its productive function as a way to falsely indicate the

possession of skill. The market recognizes this possibility and forms expectations on the extent of skill accumulation, P_s.

Based on this assessment, the skilled workers may limit their participation in firms of unrecognized productivity, to reduce the potential for adverse selection, hence the incidence of holdup problems. This further lowers the return from investing in skill, as it lessens the equilibrium probability to match with an equally productive worker $P_s\left[\gamma + (1-\gamma)\omega\right]$. This expression captures the endogenous marketability of skill, being affected not only by the informational friction γ, but also by the strategies $\{\delta, \sigma, \omega\}$.

We are now ready to state the following:

Definition. *A symmetric stationary equilibrium is a list of strategies* (δ, σ, ω)*, and value functions* (V_n, V_d, V_s) *that satisfy (3)-(6).*

Before proceeding with discussing the possible outcomes, we note that in equilibrium $\sigma > 0$ only if $V_s > V_d$. That is, in equilibrium agents acquire skill only if the more productive have the largest lifetime return from marketing their labor. This is a desirable feature of the model, because $V_s > V_d$ captures the empirical finding that although remuneration schemes do not appear to sufficiently reward educational achievement and greater productivity in the short run, they do so in the long-run (e.g. Bishop, 1987, 1991, Lazear, 1977).

5 Existence and Characterization of Equilibria

Expression (3) underscores that a necessary condition for skill accumulation is $c_d \leq \max(V_s) - c_s - V_n$. In short, schooling costs cannot exceed the largest possible benefit from earning skills. From (6) we see V_s is a maximum when $P_s = 1$ and $\omega = 1$, in which case $r\left(V_s - c_s - V_n\right) = G_s(s) - \phi$, where

$$\phi = u_n - (u_s - rc_s).$$

Thus, skill accumulation is feasible only if $c_d \leq \bar{c}$ where

$$\bar{c} = \frac{G_s(s) - \phi}{r}.$$

Hence, in the remainder of the paper we retain the assumption $\bar{c} > 0$, that is

$$G_s(s) > \phi. \tag{7}$$

Consequently, a planner would make everyone invest in education *and* skill if $c_d \leq \bar{c}$. To see why, recall that the most surplus is generated by skilled teams (see (1)). When (7) is in place, then $c_d \leq \bar{c}$ implies the net present value of the stream of productivity gains, $\frac{G_s(s)+u_s}{r} - (c_d + c_s)$, overtakes the opportunity cost u_n. Thus, $P_s = 1$ is socially desirable.

To more readily compare the equilibrium outcomes to the social optimum, we classify all possible equilibria according to the distribution of skill/degrees they can sustain. We use the notation $P_s^* \in (0,1)$ to indicate an economy where workers have heterogeneous productivity. This may occur for two reasons. It is possible that some agents may simply avoid education, while those who go to school do earn skill. Here, $P_d = 0$ so that skill and schooling are perfectly correlated. It is also possible that some students avoid investing in skill, hence schooling and skill are imperfectly correlated, i.e. $P_d = P_d^* \in (0,1)$. The next table summarizes this discussion:

	$P_d = 0$	$P_d = P_d^*$	$P_d = 1$
$P_s = 0$	unskilled workforce		
$P_s = P_s^*$	heterogeneous workforce & skilled educated workers	educated workers are heterogeneous	
$P_s = 1$	skilled workforce		

5.1 Equilibria with Full Observability

It is useful to study the case $\gamma = 1$ to identify the sources of incentives to skill accumulation, absent adverse selection problems. Here, degrees cannot falsely indicate the possession of skill. Thus, skilled agents avoid cross matches and so education is undertaken only as a productive endeavor. Individual incentives to do so exist if—given some market-compensation—skills are sufficiently marketable. This is subject to strategic uncertainty as educational choices are not coordinated among agents. Hence—as is standard—there is a strategic complementarity that generates coexistence of equilibria, some of which are market failures. Thus, the outcome of the model with $\gamma = 1$ reflects the predictions of a model in which education is productive and degrees are a perfect signals of skill.

To formalize these considerations we look at the agent's best responses. A student prefers

earning skills and going to school if, reformulating (2) and (3),

$$P_s G_s(s) \geq \phi \tag{8}$$

$$P_s G_s(s) \geq \phi + r c_d. \tag{9}$$

The left-hand-sides of (8)-(9) measure the *market incentive* to earn skill, which grows with the number of good jobs, P_s, and with the skill remuneration $G_s(s)$. The right-hand-side measures the incentives linked to mere autarkic production. It can be interpreted as the opportunity cost from sitting in autarky unskilled—as in (9)—or with a useless degree—as in (8). Two features stand out. If (9) holds, then so does (8). Hence education is always productive, i.e., $P_d = 0$, because the market only remunerates skills. Second, if $\phi < 0$, then the market is an *irrelevant* source of incentives to student effort. The reason is that skill increases autarkic productivity so much, that autarkic production gains *alone* justify earning skill.

More formally, if we define the constant

$$\underline{c} = \max(0, \frac{-\phi}{r})$$

we can then discuss equilibrium existence (all proofs are in appendix).

Proposition 1. *Suppose $\gamma = 1$. We have the following symmetric equilibria:*

i. If $c_d > \bar{c}$, then $\delta = 0$ is the unique equilibrium.

ii. If $c_d \in [\underline{c}, \bar{c}]$, then three equilibria coexist. One with $(\delta, \sigma) = (1, 1)$, one with $\delta = 0$, and one with $\sigma = 1$ and $\delta = \delta^* = \frac{\phi + r c_d}{G_s(s)}$.

iii. If $c_d < \underline{c}$, then $(\delta, \sigma) = (1, 1)$ is the unique equilibrium.

Hence, the following equilibrium distributions of skill and degrees can arise:

$$(P_s, P_d) = \begin{cases} (1, 0) & \text{if } c_d \leq \bar{c} \\ (P_s^*, 0) & \text{if } \underline{c} \leq c_d \leq \bar{c} \\ (0, 0) & \text{if } c_d \geq \underline{c}. \end{cases}$$

In short, when $\gamma = 1$ a degree is a perfect signal of skill. In this case education is always productive—and so is perfectly correlated with skill. Although there is no gain from exiting a

school unskilled, schooling decisions *do* depend on education costs. When $c_d > \bar{c}$ the net return from education is negative, so no one goes to school. The opposite holds if education is very cheap; if $c_d < \underline{c}$ then $P_s = 1$. At moderate costs, $c_d \in [\underline{c}, \bar{c}]$, equilibria with and without skill heterogeneity coexist if—generally speaking—skill accumulation depends on market incentives. This originates from a strategic complementarity in private investment in education.

To see why, Figure 1 plots P_s against c_d. In panel (a) $\phi = 0$, so that autarky is not a source of bias in favor or against the acquisition of skills. A coordination failure with no skill accumulation can always arise since individual schooling decisions hinge on the aggregate skill level P_s. Raising own productivity is optimal when $P_s = 1$ since every match is desirable. As P_s falls, so does the probability of a desirable match, and the incentive to earn skill falls. When $P_s = P_s^*$ agents randomize—as possession of skill generates zero net returns—while education is not undertaken if $P_s < P_s^*$.[7] Clearly, $\frac{\partial P_s^*}{\partial c_d} > 0$ since in a mixed strategy equilibrium higher schooling costs require more good matches.

To demonstrate how ϕ affects the equilibrium consider panels (b) and (c). When $\phi < 0$, skill guarantees productivity gains even in autarky. Thus, the model is biased *in favor* of the acquisition of skill, which is why $P_s = 1$ uniquely arises when education is cheap. Instead, the model is biased *against* the acquisition of skills when $\phi > 0$, and $P_s = 1$ never arises uniquely. In short, we tend to see strategic complementarities in individual choices when they hinge on market-based incentives, i.e. when $\phi \geq 0$. In this case, three equilibria always coexist. How do they compare?

To answer this question we consider the standard ex-ante welfare criterion

$$rW(P_s, P_d) = P_s r(V_s - c_s - c_d) + P_d r(V_d - c_d) + (1 - P_s - P_d) r V_n.$$

Note that if $c_d \leq \bar{c}$, then we have

$$\begin{aligned} rW(1,0) &= G_s(s) + u_s - rc_s - rc_d \\ &= u_n + G_s(s) - (\phi + rc_d) \\ &> rW(P_s^*, 0) = rW(0,0) = u_n \end{aligned} \tag{10}$$

since $c_d \leq \bar{c}$ corresponds to $G_s(s) \geq \phi + rc_d$. The outcome $\delta = \sigma = 1$ is socially desirable because skilled matches generate the greatest surplus, and because $P_s = 1$ maximizes the incidence of such

[7] This is reminiscent of Snower's (1996) "low-skill, bad-job traps." In countries where few good jobs are available workers have little incentive to acquire skills. This behavior feeds back on firms' ability to provide good jobs.

matches. However, coordinating on this outcome is difficult because individual choices are made in isolation. Hence, as is well known, coordination failures may occur.

5.2 Equilibria with Imperfect Observability

We now set $\gamma < 1$ to study the possibility of free-riding behavior. This depends not only on the magnitude of informational frictions, but also on education costs and the relative market remuneration of skill, $G_s(s)/G_d(s)$. This is evident when checking the counterparts to (8) and (9). A student prefers to exploit education's productive role if, from (2):

$$P_s\left[\gamma + (1-\gamma)\,\omega\right]^2 G_s(s) + P_d(1-\gamma)\omega G_s(d) - P_s(1-\gamma)\omega G_d(s) \geq \phi \tag{11}$$

while, from (3), investing in schooling is optimal if:

$$P_s\left[\gamma + (1-\gamma)\,\omega\right]^2 G_s(s) + P_d(1-\gamma)\omega G_s(d) \geq \phi + rc_d. \tag{12}$$

Relative to the perfect informational case, the key similarity is that student achievement always hinges on market-based incentives if $\phi \geq 0$. Thus, for simplicity we focus on $\phi = 0$, when there is no explicit bias in favor or against the acquisition of skill.

The key departure from the case $\gamma = 1$ is that students can now be indifferent to earning skill, because a degree now provides an imperfect signal of skill. Consequently, educated workers may have heterogeneous skill in equilibrium, $P_d = P_d^*$. To understand why, note that (11) can hold with equality, when (12) is satisfied. This will happen when the return from marketing skills—represented by the LHS of (12)—is moderate. The reason is that students exert effort to earn skill based solely on the expected skill premium. This is simply the difference between the return from marketing skills or just a degree—the LHS of (11). Substantial informational frictions (low γ) or poorly remunerated productivity (low $\frac{G_s(s)}{G_d(s)}$) decrease the skill premium, hence may cause indifference to skill, so agents set $\sigma = \sigma^*$.

To formalize this intuition we define the critical values

$$\begin{aligned}
\underline{\gamma} &= 1 - \tfrac{G_s(s)}{G_d(s)} \\
\bar{\gamma} &= \tfrac{G_d(s)}{G_d(s)+G_s(s)} \\
c_L(\gamma) &= \tfrac{-r^{-1}\gamma^2 G_d(s)G_s(d)G_s(s)}{-G_s(d)[G_d(s)-\gamma G_s(s)]+\gamma G_s(s)G_d(s)} \\
c_M(\gamma) &= \tfrac{-r^{-1}(1-\gamma)^2 G_d(s)G_s(d)}{G_s(s)-(1-\gamma)[G_s(d)+G_d(s)]}
\end{aligned}$$

which we use in discussing equilibrium existence, below. Here we note that $\underline{\gamma} < \bar{\gamma} \leq 1$, $0 < c_L(\gamma) \leq c_M(\gamma)$ if $\gamma \leq \bar{\gamma}$, and $c_M(\gamma) \leq \bar{c}$ if $\underline{\gamma} \leq \gamma \leq \bar{\gamma}$. It follows

Proposition 2. *Suppose $\gamma < 1$ and $\phi = 0$. We have the following symmetric equilibria:*

i. $(\sigma, \delta) = (0, 0)$ always; it is unique if $c_d > \bar{c}$, or if $c_d > c_L(\gamma)$ and $\gamma < \underline{\gamma}$.

ii. If $c_d \leq \bar{c}$ and $\gamma \geq \underline{\gamma}$, then $(\sigma, \omega) = (1, 1)$ and $\delta = 1$ or $\delta = \delta^* = \frac{rc_d}{G_s(s)}$.

iii. If $c_d \leq c_M(\gamma)$ and $\underline{\gamma} \leq \gamma \leq \bar{\gamma}$, then $(\sigma, \omega) = (\sigma^*, 1)$ and $\delta = 1$ or $\delta = \delta^*$ with

$$\sigma^* = \frac{-(1-\gamma)G_s(d)}{G_s(s)-(1-\gamma)G_d(s)-(1-\gamma)G_s(d)} \quad \text{and} \quad \delta^* = \frac{rc_d}{(1-\gamma)G_d(s)} \times \frac{1}{\sigma^*}$$

iv. If $c_d \leq c_L(\gamma)$ and $\gamma \leq \bar{\gamma}$, then $(\sigma, \omega) = (\sigma^*, \omega^*)$ and $\delta = 1$ or $\delta = \delta^*$ with

$$\omega^* = \frac{\gamma^2 G_s(s)}{(1-\gamma)[G_d(s)-\gamma G_s(s)]}, \quad \sigma^* = \frac{-\gamma G_s(d)}{-\gamma G_s(d)+(1-\gamma)G_d(s)\omega^*} \quad \text{and} \quad \delta^* = \frac{rc_d}{(1-\gamma)G_d(s)} \times \frac{1}{\sigma^*\omega^*}$$

Hence, the following equilibrium distributions of skill and degrees can arise:

$$(P_s, P_d) = \begin{cases} (P_s^*, 0) \text{ or } (1, 0) & \text{if } c_d \leq \bar{c} \text{ and } \gamma \geq \underline{\gamma} \\ (P_s^*, P_d^*) & \text{if } c_d \leq c_M(\gamma) \text{ and } \gamma \leq \bar{\gamma} \\ (0, 0) & \text{always.} \end{cases} \tag{13}$$

The key result is that now the economy can end up with an educated workforce of low average skill, as when $P_d = P_d^*$. Of course, this is not the only equilibrium, since outcomes with different patterns of schooling and skill accumulation (P_s, P_d) generally coexist. Rather, now that $\gamma < 1$ we have a greater richness of equilibria. This is illustrated in Figure 2, drawn for the case $G_s(s) < G_d(s)$,[8] where we have

$$(P_s, P_d) = \begin{cases} (P_s^*, 0) \text{ or } (1, 0) & \text{in areas 3, 4, and 5} \\ (P_s^*, P_d^*) & \text{in areas 2, 3, and 4} \\ (0, 0) & \text{everywhere.} \end{cases}$$

Before discussing the various equilibria, we compare them in terms of ex-ante welfare.

5.2.1 Ex-Ante Social Welfare

[8] The figure with $G_s(s) \geq G_d(s)$ is similar but areas 1 and 2 are incorporated into areas 4 and 3, respectively. The key difference is $(P_s, P_d) = (0, 0)$ arises *uniquely* in area 1 only if $G_s(s) < G_d(s)$, i.e. if skill is poorly compensated.

Start by recalling that if $c_d \leq \bar{c}$, then $P_s = 1$ in the social optimum. Once again, this is because mismatched worker types generate less surplus than skilled matches. Naturally, under-investment in skill generates social inefficiencies.

Corollary 3. *Suppose $\gamma < 1$ and $\phi = 0$. Consider $c_d \leq \bar{c}$. Then, ex-ante welfare $rW(P_s, P_d)$ is (i) maximized when $(P_s, P_d) = (1, 0)$, corresponding to the social optimum, (ii) achieves the minimum, u_n, whenever $P_d < 1 - P_s^*$, and (iii) achieves an intermediate value whenever $P_d = 1 - P_s^*$.*

The main difference with the case $\gamma = 1$, is that under-accumulation of skill does not necessarily imply the same level of social (in)efficiency experienced when every worker is unskilled. The reason is that now degrees cannot clearly reveal skill. Hence, equilibria may exist where the more productive workers can be 'held-up' by partners that are not recognized as being less productive. If free-riding off the skills of others is easy and relatively profitable, incentives exist to go to school minimizing the study effort, rather than remaining unskilled *and* without a degree. This, of course, generates inefficiencies, which hinge on several margins.

There is always a negative *extensive* margin effect, since any equilibrium with heterogeneous education does not maximize the surplus-generating matches. An additional negative *intensive* margin effect may arise when educated workers are of heterogeneous skill, if the surplus from a cross-match is negative.[9]

Can we avoid such inefficiencies? To answer this question we need to understand how the three central elements of the analysis—informational frictions, education costs, and market remuneration of skill—impinge on the possible equilibrium outcomes.

5.2.2 informational Frictions

To focus on the role of informational frictions, it is helpful to refer to Figure 2, choosing a value for c_d and moving along an imaginary vertical line. We start by observing that absence of education is always an equilibrium. So, let us focus on the other possible equilibria.

If $\gamma > \bar{\gamma}$ then a degree conveys information about a worker's productivity rather well. Con-

[9] The planner would not necesarily force skilled agents to cross-match. Whether $G_d(s) + G_s(d)$ is positive or not, forcing $\omega = 1$ increases the incidence of hold-up problems. Thus, forcing skilled agents to cross-match always creates inefficiencies along some margin, intensive or extensive.

sequently, every student *chooses* to make education a productive endeavor, and the educated workforce is homogeneously skilled ($P_d = 0$ in area 5). Consequently, the possible outcomes are as in Proposition 1.

As γ falls in the range $[\underline{\gamma}, \bar{\gamma}]$ informational frictions are more pronounced, degrees become even more imperfect signals of skill, and a richer typology of outcomes arises. Depending on education costs, students may under-invest in skill (areas 3, 4) or not (area 5). In particular, all possible equilibria coexist when education is cheap (area 3).

When $\gamma < \underline{\gamma}$, degrees are so uninformative signals of skill that either no-one goes to school (the unique outcome in area 1) or if someone does, then a fraction of the students always under-invests in skill (area 2).

Under-investing in skill simply means that some students set $\sigma = \sigma^*$; they free-ride by minimizing study effort, in order to earn a degree, but not skill. Thus, education and skill are imperfectly correlated in equilibrium ($P_d = P_d^*$). This behavior sustains two types of heterogeneity, depending on the strategy δ. We can have a fully educated workforce of heterogeneous skill, $\delta = 1$, or a partially schooled workforce where educated workers are unequally productive, $\delta = \delta^*$. That is, either we have $P_d^* = 1 - P_s^*$, or we have $P_d^* < 1 - P_s^*$.

The crucial observation is equilibria where the educated workforce has low average skill arise only if informational frictions are substantial. The cause is the possibility of holdup problems, as skilled workers may unknowingly end up in cross-matches where they lose some productivity rents. This creates incentives to under-invest in skill, thus reducing the overall level of human-capital. The incidence of holdup problems increases with the severity of informational frictions, as the potential for adverse selection in matching grows. This is why $P_d = P_d^*$ arises only for $\gamma \leq \bar{\gamma}$. As γ falls below $\underline{\gamma}$, the potential for adverse selection is so dire that skilled workers limit their participation in firms of uncertain productivity, setting $\omega = \omega^*$ (areas 2, 3).[10]

Thus, our model formalizes the notion that there are less incentives to accumulate human capital when academic certificates are indistinct yardsticks of achievement, than when they are not. Thus, two education policy guidelines seem to suggest themselves. First, a primary concern of the educational system should be to be foster academic achievement. Second, the educational system should provide meaningful degrees. To give an example, consider high-school education in

[10] Obviously, $\omega = 0$ is inconsistent with $P_d > 0$, as unskilled workers could never earn rents on the market.

the U.S. There is evidence that employers pay little attention to grades, perhaps because of limited informational content (Owen, 1995). For instance, Bishop (1988, 1990) argues that employers do not rely on high-school transcripts in making hiring decisions, but simply on possession of a diploma. Then, if letter grades are meant to measure human capital accumulation, our model suggests that grade inflation or poor testing procedures are undesirable, as they impair the market's ability to use academic certificates to assess productivity. Policies directed at increasing the informative content of a diploma—perhaps through standardized testing—seem desirable.[11] This intuition also applies to college education as the recent hotly debated issue of U.S. college grade inflation seems to suggest (e.g. see the recent article by Hedges, 2004).

5.2.3 Schooling Costs and the Remuneration of Skill

What role do economic incentives have on the private decision to (under)invest in skill? Figure 3 traces the values P_s associated to Figure 2, when $\gamma \in (\underline{\gamma}, \bar{\gamma})$. The lines through 0 and 1 identify $P_s = 0, 1$, and the others refer to $P_s = P_s^*$. Specifically, $P_s^*(H)$ corresponds to $(P_s, P_d) = (P_s^*, 0)$. The remaining curves identify episodes of skill inequality across educated workers, $(P_s, P_d) = (P_s^*, P_d^*)$; here two outcomes coexist, depending on whether everyone has a degree or not.[12] Clearly, lower values of c_d admit outcomes with lower steady state skill levels, which is what we formalize in the following:

Corollary 4. *Suppose $\gamma < 1$ and $\phi = 0$. Equilibria where students under-invest in skill, $P_d = P_d^*$, and in which the skill accumulation level P_s is the lowest, arise when the cost of education c_d and the relative market remuneration of skill $G_s(s)/G_d(s)$ are low.*

This result hinges on two features of the model. First, free-riding occurs in economies where degrees are not only uninformative signals of skill, but are *also* easy to get. This follows from the observation that for $\gamma \leq \bar{\gamma}$, then only if $c_d \leq c_M(\gamma)$ we can have $P_d = P_d^*$. It is obvious that everyone would want an inexpensive degree in our model, as $G_d(s) > 0$ reflects a fundamental

[11] Masters (2004) develops a matching model in which workers are ex-ante heterogenous and their productivity is private information. Firms give an employment test that is assumed to be less accurate for a subset of the population, called "ethnic minority." Greater test accuracy slows down the matching rate for everyone leading to higher levels of unemployment.

[12] The flat segments correspond to $P_d^* = 1 - P_s^*$. Note that $P_s^*(H)$ is below any other P_s^* since in that equilibrium no student free rides. Hence, incentives to earn skill arise despite a lower probability of skilled matches.

inability to screen out undesirable partners by means of appropriate contracts. All else equal, lower education costs raise the incentive to get a degree while avoiding the additional effort required to earn productive skill. Consequently, the lower is the informativeness of degrees, the smaller is the barrier represented by education costs. This is why as γ falls $c_M(\gamma)$ increases in Figure 2.

Second, the parameter space that supports equilibria where students under-invest in skill shrinks as the relative market remuneration of skill increases. This follows from the observation that $\underline{\gamma}$ and $\bar{\gamma}$ drop as $\frac{G_s(s)}{G_d(s)}$ rises. All else equal, students' incentives to free-ride are weak if skills are well compensated. This is true even if informational frictions are substantial, since $\bar{\gamma} \to 0$ in the limit as $\frac{G_s(s)}{G_d(s)}$ grows large. The opposite holds when skills are poorly compensated, which reduces the opportunity cost of minimizing study effort. In fact, this might eliminate altogether the desire to get an education, as when skill is so ill-rewarded on the market, that $\underline{\gamma} > 0$. In this scenario, when $\gamma < \underline{\gamma}$ then absence of education is the unique outcome even if education has a moderate cost, i.e., $c_d > c_L(\gamma)$ (area 1 in Figure 2).

The analysis suggests that improving the affordability of education may be counterproductive if incentives for academic achievement are limited and academic certificates are vague productivity measures. Some observers indicate these are features seen in U.S. high-school education. For example, the Commission on the Skills of the American Workforce (1990) indicated that employers have realized that it is possible to graduate from U.S. high schools and still be functionally illiterate, and noted that:

> "Many employers require a high school diploma for all new hires, yet very few believe that the diploma indicates educational achievement. ... [T]he non college bound know that their performance in high school is likely to have little or no bearing on the type of employment they manage to find."

Our model suggests that if the productivity of education depends on effort while in school, then an emphasis on student achievement must necessarily complement attempts directed at making education more affordable. In fact, focusing attention solely on subsidization of private schooling costs may be an ineffective way to improve the average skill level, in the long run. These considerations are particularly relevant when skills earned in school are poorly compensated, because in that scenario market incentives to student achievement are the weakest.

Several studies concerning education financing and reform have emphasized the importance of high standards in increasing average skill levels (e.g. Betts, 1998, and Costrell, 1994). Our analysis indicates that high standards can be even more critical when the private cost of education is low. This suggests implications for tuition subsidy policies. For example, having a government pay a larger share of the private cost of college education, in our environment, simply amounts to increasing the incentive to minimize study effort. In turn, this would simply provide less incentives to accumulate human capital, not more. This intuition complements a recent calibration exercise proposed by Sahin (2003) who finds that subsidizing tuition increases enrollment rates but reduces student effort, hence human capital accumulation.[13]

The failure of subsidies to improve educational outcomes is something our model shares with "pure signaling" models of education, i.e. models where agents are ex-ante heterogeneous and education has no productive role. There, subsidies generally facilitate occurrence of a pooling equilibrium in which the unskilled mimic those more productive. In our model, however, differences in ability among the educated workers are a direct result of insufficient *incentives* to achievement in school, rather than innate skill heterogeneity. We note that if we modified our model by introducing ex-ante heterogeneity, and let education have only a signaling role for those with innate skills, the main result would still emerge. Equilibria would arise in which agents underinvest in skill, as the more productive agents face a holdup problem. In fact, we think that the incentives to earn skill would be even lower than in our current formulation.

To see why, suppose some agents are ex-ante skilled and some not. Suppose the unskilled can go to school to earn skill, while the skilled can go to school just to signal their innate abilities. There would be incentives to match with those who have no degree, if the proportion of innately productive agents is large. There would be disincentives to use education as a signal of innate productivity, because the market remuneration of the skilled suffers from contractual imperfections. Thus, we expect that less of the unskilled agents would find it worthwhile to exploit the productive

[13] Unlike our model, Sahin models parents as making education choices and children (who differ in intellectual ability and motivation) making effort choices. Thus—unlike our model—skill heterogeneity hinges on ex-ante differences. Tuition subsidization adversely affects incentives to achievement in two ways. All students reduce study effort with lower tuition levels (as in our model). A low-tuition, high-subsidy policy causes an increase in the ratio of less able and less motivated students among college graduates (unlike our model where there is no ex-ante heterogeneity).

role of education, if a sufficient population proportion is innately more productive. This effect would be magnified if education had no productive role. Of course, in this pure signaling story, the composition of skill in the economy would be invariant to policy, being unaffected by changes in the cost of education or informational frictions (although the schooling decisions could be affected). This is very different than our model, in which the composition of skill in the workforce is endogenous.

6 Final Considerations

We have built a simple model where education's productive role is endogenous, as skill accumulation is a decision that is *complementary* to that of educational investment. Market failures can arise, which are characterized by pervasive education but under-investment in skill. This result hinges on the existence of two frictions in our model. First, degrees only certify the undertaking of the educational opportunity but provide vague information on student achievement. Second, contractual imperfections create holdup problems that redirect productivity rents to the less productive workers.

To the extent that these frictions are relevant features of field economies (and there is reason to believe they are[14]), our study has several implications for education policy. A key concern of the educational system should be the provision of incentives to student achievement. Especially, the analysis suggests that an increased public effort to raise enrollment by lowering the private cost of education may be ineffective in improving the workforce's skills when not complemented by incentives to student performance. In fact, we have demonstrated that when incentives to student performance are weak some policies that are successful in raising enrollment may have negative consequences on educational outcomes and aggregate productivity. If students' motivation to achievement is weak, such policies support equilibria where it is individually optimal to earn a degree while choosing to accomplish little. Improving the quality of education is a more effective policy, because it raises the expected return from schooling. In short, the debate on education financing cannot be separated from that of education reform.

[14] Owen (1995) discusses contributions from economics and other social sciences, devoting attention to the relationship between cognitive achievement and labor market productivity, incentives to achievement, and public policy. Hanushek (1986) surveys analyses of the educational process and their policy implications.

The study leads also to interesting parallels about the possible role of technological change favoring skilled workers in explaining the increase in wage inequality experienced in the U.S. (e.g. Bound and Johnson, 1992, Katz and Murphy, 1992). Our model suggests that the increase in wage inequality within education groups could be the rational response of the market to the perception that educational certificates are poor signals of productivity. In this scenario, increasing the relative remuneration to skill can be seen as an attempt at bypassing the educational sector's inability to provide students with sufficient incentives to maximize their educational attainment. This interpretation could be complementary, perhaps, to the skilled-biased technological change explanation for the increase in within-group inequality offered in the literature.

Because of the simplicity of the model, ours is clearly not meant to be a comprehensive study of education's role in promoting human capital accumulation. However, this simplicity is not a liability as the key results appear to be robust to richer specifications that preserve the following features: (i) human capital accumulation hinges on student effort, (ii) a worker's productivity is observed imperfectly in the early stages of a worker's career, and (iii) workers known to be more productive are better compensated, i.e. skill commands a premium (see Blankenau and Camera, 2004). In fact, we think the approach adopted can provide a useful conceptual framework in developing intuition about the ramifications of endogenizing education's productive role.

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Appendix

Proof of Proposition 1. To prove existence of equilibria, we use a constructive approach. Given a set of candidate strategies, we find parameter regions such that the strategies are individually optimal.

Let a superscript '*' identify a variable in the open unit interval, i.e. $\sigma^* \in (0,1)$ etc. From (2) and (3) it follows that

$$\sigma' = \begin{cases} 1 & \text{if } \delta\sigma G_s(s) > \phi \\ [0,1] & \text{if } \delta\sigma G_s(s) = \phi \\ 0 & \text{if } \delta\sigma G_s(s) < \phi \end{cases} \tag{14}$$

$$\delta' = \begin{cases} 1 & \text{if } \delta\sigma G_s(s) - rc_d > \phi \\ [0,1] & \text{if } \delta\sigma G_s(s) - rc_d = \phi \\ 0 & \text{if } \delta\sigma G_s(s) - rc_d < \phi \end{cases} \tag{15}$$

We study the set of symmetric equilibria $(\delta', \sigma') = (\delta, \sigma)$ for different values of c_d.

- Start by proving that if $c_d \geq \bar{c}$ then $\delta = 0$ in *any* equilibrium (which implies $P_d = P_s = 0$). To see why, conjecture $(\sigma, \delta) \in [0,1]^2$. If $c_d \geq \bar{c} = \frac{G_s(s)-\phi}{r}$, then from (15) we have $\delta' = 0$. Thus $\delta' = \delta = 0$ is the unique symmetric equilibrium.

- Prove that if $c_d \geq \underline{c} = \max(0, \frac{-\phi}{r})$, then $\delta = 0$ is an equilibrium (which implies $P_d = P_s = 0$). Notice that $\underline{c} < \bar{c}$. To do so conjecture $\delta = 0$ and consider $c_d \geq \underline{c}$, which may be because $c_d \geq \underline{c} > 0$ (i.e. $\phi < 0$) or because $c_d > 0 \geq \underline{c}$ (i.e. $\phi \geq 0$). Since $c_d \geq \underline{c}$, then (15) implies $\delta' = 0$. The strategy σ is irrelevant in equilibrium.

- Prove that if $c_d \leq \bar{c}$ then $(\sigma, \delta) = (1,1)$ is always an equilibrium (this implies $P_s = 1$). Conjecture $\delta = \sigma = 1$. From (14) we have $\sigma' = 1$ since we have assumed $G_s(s) > \phi$. From (15) we see that if $c_d \leq \bar{c}$ then $\delta' = 1$.

  Note that if $c_d < \underline{c} < \bar{c}$ then $(\sigma, \delta) = (1,1)$ is the *unique* equilibrium. To see why consider $\phi < 0$, hence $\underline{c} > 0$. If $c_d < \underline{c}$ expression (15) tells us $\delta = 1$ is the unique symmetric strategy. Since $\phi < 0$ then $\sigma = 1$ is the unique symmetric skill-accumulation strategy, from (14).

- Prove that if $c_d \in [\underline{c}, \bar{c}]$ then $\sigma = 1$ and $\delta = \frac{\phi + rc_d}{G_s(s)}$ is an equilibrium. Suppose $\sigma = 1$ and $\delta = \delta^*$ is an equilibrium. Then (15) implies $\delta^* = \frac{\phi + rc_d}{G_s(s)}$ is the unique symmetric equilibrium

mixed strategy. Notice that $\delta^* \in (0,1)$ only if $c_d \in [\underline{c}, \bar{c}]$. Plugging $\sigma = 1$ and $\delta = \frac{\phi + rc_d}{G_s(s)}$ into (14) we observe that $\sigma' = 1$ is indeed a best response since $rc_d > 0$.■

We now present a set of results that will be used in the next set of proofs.

Lemma A. *Define*

$$\sigma_\omega = \frac{-G_s(d)(G_d(s)-\gamma G_s(s))}{-G_s(d)(G_d(s)-\gamma G_s(s))+\gamma G_s(s)G_d(s)} = \frac{-\gamma G_s(d)}{-\gamma G_s(d)+(1-\gamma)G_d(s)\omega^*(\gamma)}, \quad \sigma_1 = \frac{-G_s(d)(1-\gamma)}{G_s(s)-(1-\gamma)(G_d(s)+G_s(d))}$$
$$\omega^* = \frac{\gamma^2 G_s(s)}{(1-\gamma)(G_d(s)-\gamma G_s(s))}$$

We have:

1. *If* $\gamma \in \left[\underline{\gamma}, \bar{\gamma}\right]$ then $\bar{c} \geq c_M(\gamma)$; further, $c_M(\gamma) \geq c_L(\gamma)$ *whenever* $\gamma \leq \bar{\gamma}$.

2. $0 < \sigma_1 \leq \sigma_\omega < 1$ *if and only if* $\gamma \leq \bar{\gamma}$ *and* $G_s(s) \geq G_d(s)$. *Furthermore,* σ_ω *and* σ_1 *are both decreasing in* γ, *whereas, if* $\gamma \leq \bar{\gamma}$, *then* ω^* *is increasing in* γ.

Proof. Note that $\underline{\gamma}$ and $\bar{\gamma}$ are decreasing in $G_s(s)/G_d(s)$, and $\underline{\gamma} < \bar{\gamma} \leq 1$. Consider $\gamma \leq \bar{\gamma}$.

1. It is a matter of algebra to show that $c_L(\gamma) \leq c_M(\gamma)$. Now consider $\bar{c} - c_M(\gamma)$, which is strictly increasing in γ, and such that $\bar{c} - c_M(1) > 0$. Also, $\bar{c} - c_M(0) \geq 0$ whenever $G_s(s) \geq G_d(s)$. Conversely, if $G_s(s) < G_d(s)$ then $\bar{c} - c_M(\gamma) \geq 0$ only if $\gamma \geq \underline{\gamma}$. Since $\bar{\gamma} > \underline{\gamma}$, then $\bar{c} \geq c_M(\gamma)$ if $\gamma \in \left[\underline{\gamma}, \bar{\gamma}\right]$.

2. To show $0 < \sigma_1 \leq \sigma_\omega < 1$, consider the case where $\sigma_\omega, \sigma_1 > 0$. Rearrange $\sigma_\omega < \sigma_1$ as $\gamma\left(G_d(s) - G_s(s)\right) < \bar{\gamma}\left(G_d(s) - G_s(s)\right)$. Then, if $\gamma \leq \bar{\gamma}$, we have $\sigma_\omega < \sigma_1$ when $G_s(s) < G_d(s)$, and $\sigma_\omega \geq \sigma_1$ otherwise. It can also be shown that σ_ω and σ_1 are decreasing in γ, and ω^* is increasing in γ when $\gamma \leq \bar{\gamma}$.■

Proof of Proposition 2. Let $\phi = 0$. Recall the definition $\bar{c} = G_s(s)r^{-1}$. We consider the different equilibria, by looking at all possible combinations of σ, δ and ω.

1. Prove $\delta = 0$ and $(\sigma, \omega) = (0,0)$ is always an equilibrium.
   Conjecture $\sigma = \delta = 0$. Then $\omega' = \omega = 0$, from (4), in a symmetric equilibrium. Hence, from (11) we have $\sigma' = 0$. From 3 and 6, we also have $\delta' = 0$. Here $(P_s, P_d) = (0,0)$.

2. Prove $\delta \in \{\delta^*, 1\}$ and $(\sigma, \omega) = (1, 1)$ is an equilibrium if $c_d \leq \bar{c}$ and $\gamma \geq \underline{\gamma}$.

   Conjecture $\sigma = \omega = 1$ and $\delta > 0$. From (4) we have $\omega' = 1$. From (11) we have $\sigma' = 1$ *only if* $\delta\left[G_s(s) - (1-\gamma)G_d(s)\right] > 0$, which—given $\delta > 0$—requires $\gamma > \underline{\gamma}$. Under the conjectured equilibrium, from (12) we have $\delta' > 0$ if $\delta G_s(s) - rc_d \geq 0$ i.e. if $\delta \geq \delta^* = \frac{rc_d}{G_s(s)} \in (0, 1]$. Thus, if $G_s(s) > rc_d$ (i.e. $c_d < \bar{c}$) then we have $\delta = 1$ or $\delta = \delta^*$ as possible equilibrium strategies. Here, we either have $(P_s P_d) = (P_s^*, 0)$ or $(P_s P_d) = (1, 0)$.

3. Finally, prove (i) $(\sigma, \omega) = (\sigma^*, \omega^*)$ and $\delta \in \{\delta^*, 1\}$ if $c_d \leq c_L(\gamma)$ and $\gamma \leq \bar{\gamma}$; (ii) $(\sigma, \omega) = (\sigma^*, 1)$ and $\delta \in \{\delta^*, 1\}$ if $c_d \leq c_M(\gamma)$ and $\underline{\gamma} \leq \gamma \leq \bar{\gamma}$.

   Conjecture $\delta > 0$, $\sigma = \sigma^*$ and $\omega > 0$. From (11) we have $\sigma' = [0, 1]$ if

$$\frac{\gamma\delta\sigma}{1-\gamma}\left[\gamma + \omega(1-\gamma)\right]G_s(s) + \omega\left\{\delta\sigma\left[\gamma + \omega(1-\gamma)\right]G_s(s) + \delta(1-\sigma)G_s(d)\right\} = \delta\sigma\omega G_d(s) \tag{16}$$

   that is if $V_d = V_s - c_s$. Using (12) and $V_d = V_s - c_s$, we have that $\delta' > 0$ requires

$$\delta\sigma\omega(1-\gamma)G_d(s) \geq rc_d \tag{17}$$

   Thus, in a symmetric equilibrium $\delta = 1$ if (17) holds with strict inequality, and $\delta = \delta^*$ otherwise. Using (4) we have that $\omega' > 0$ if

$$\sigma\left[\gamma + (1-\gamma)\omega\right]G_s(s) \geq -(1-\sigma)G_s(d). \tag{18}$$

   Again, in a symmetric equilibrium $\omega = 1$ if (18) holds with strict inequality, and $\omega = \omega^*$ otherwise. Given $\sigma = \sigma^*$, we must consider five different combinations of δ and ω.

   - Prove $\omega = \omega^*$ and $\delta = \delta^*$ is an equilibrium if $c_d \leq c_L(\gamma)$ and $\gamma \leq \bar{\gamma}$.

     Here (17) and (18) must be equalities. Solving (16), (17), and (18) we get:

$$\omega = \omega^*, \quad \sigma = \sigma_\omega \quad \text{and} \quad \delta = \delta_\omega = \frac{rc_d}{\omega^*\sigma^*(1-\gamma)G_d(s)}.$$

     Clearly, $\sigma_\omega \in (0, 1)$. It can be easily proved that $\omega^* \in (0, 1)$ if $\gamma \leq \bar{\gamma}$. Next, $\delta_\omega > 0$ always, and $\delta_\omega \leq 1$ if $\gamma \leq 1 - \frac{rc_d}{\omega\sigma G_d(s)}$, rearranged as $c_d \leq c_L(\gamma)$. It can be seen that since $\gamma \leq \bar{\gamma}$, then $G_d(s) > \gamma G_s(s)$ so the denominator of $c_L(\gamma)$ is positive, hence $c_L(\gamma) > 0$. Hence, $\sigma = \sigma^* = \sigma_\omega$, $\delta = \delta^* = \delta_\omega$ and $\omega = \omega^*$ if $c_d \leq c_L(\gamma)$ and $\gamma \leq \bar{\gamma}$. In this case $P_s = P_s^* = \sigma_\omega\delta_\omega$ and $\omega = \omega^*$ with $P_d + P_s = \delta_\omega$.

- Prove $\omega = \omega^*$ and $\delta = 1$ is an equilibrium if $c_d < c_L(\gamma)$ and $\gamma < \bar{\gamma}$.

  In this equilibrium (17) must be a strict inequality and (18) an equality. Solving the system of equations (16) and (18) we obtain $\sigma^* = \sigma_\omega$ and $\omega = \omega^*$. Hence, $\sigma_\omega, \omega^* \in (0,1)$ if $\gamma \leq \bar{\gamma}$. Finally, $\delta = 1$ if (17) holds as a strict inequality, which requires $c_d \leq c_L(\gamma)$. It follows that, given $\phi = 0$, if $c_d \leq c_L(\gamma)$ and $\gamma \leq \bar{\gamma}$ then we have $\sigma = \sigma_\omega$, $\delta = 1$ and $\omega = \omega^*$ as an equilibrium. Here, $P_s = P_s^* = \sigma_\omega$ and $\omega = \omega^*$ with $P_d + P_s = 1$.

- Prove $\omega = 0$ and $\delta > 0$ cannot be an equilibrium when $\sigma = \sigma^*$.

  By means of contradiction suppose $\delta > \omega = 0$ and $\sigma = \sigma^*$. Then, (12) implies $\delta' = 0$ and (16) implies $\sigma' = 0$, a contradiction.

- Prove $\omega = 1$ and $\delta = \delta^*$ is an equilibrium if $c_d \leq c_M(\gamma)$ and $\underline{\gamma} \leq \gamma \leq \bar{\gamma}$.

  Here (17) must hold with equality. The solutions to (16) and (17) are

$$\sigma^* = \sigma_1 \quad \text{and} \quad \delta^* = \delta_1 = \frac{rc_d[G_s(s) - (1-\gamma)(G_d(s) + G_s(d))]}{-G_s(d)(1-\gamma)^2 G_d(s)}$$

  Clearly $\delta_1 > 0$ and, if $G_d(s) > -G_s(d)$, then $\gamma > 1 - \frac{G_s(s)}{G_s(d)+G_d(s)}$ (if $G_d(s) \leq -G_s(d)$, any γ satisfies it). Next, $\delta_1 < 1$ if $c_d > 0$ small. When $\delta_1 \in (0,1)$, then $\sigma_1 \in (0,1)$ if $\gamma > \underline{\gamma}$. When $\omega = 1$ (18) must hold as a strict inequality, i.e. $\sigma_1 > \frac{-G_s(d)}{-G_s(d)+G_s(s)}$, which as seen earlier requires $\gamma \leq \bar{\gamma}$. Note that $\bar{\gamma} > \underline{\gamma} > 1 - \frac{G_s(s)}{G_s(d)+G_d(s)}$. Thus, for some $c_d > 0$ small, then $\underline{\gamma} \leq \gamma \leq \bar{\gamma}$ is sufficient for existence of this equilibrium. In particular, $\delta_1 < 1$ if $c_d \leq c_M(\gamma)$, and note that the denominator of $c_M(\gamma)$ is positive when $\gamma \leq \bar{\gamma}$. It follows that if $c_d \leq c_M(\gamma)$ and $\underline{\gamma} \leq \gamma \leq \bar{\gamma}$ then $\sigma = \sigma^* = \sigma_1$, $\delta = \delta^* = \delta_1$ and $\omega = 1$ is an equilibrium. Here $P_s = P_s^* = \sigma_1 \delta^* = \frac{rc_d}{(1-\gamma)G_d(s)}$ and $\omega = 1$ with $P_d + P_s = \delta^*$.

- Prove $\omega = 1$ and $\delta = 1$ is an equilibrium if $c_d \leq c_M(\gamma)$ and $\underline{\gamma} \leq \gamma \leq \bar{\gamma}$.

  Here, (17) and (18) must hold as strict inequalities. Using (16) we have $\sigma^* = \sigma_1$, hence $\sigma_1 \in (0,1)$ if $\gamma \geq \underline{\gamma}$. Next, (18) holds as a strict inequality if $\sigma_1 > \frac{-G_s(d)}{-G_s(d)+G_s(s)}$. Then $\gamma \leq \bar{\gamma}$ is necessary. Finally, (17) is strict if $\sigma_1 > rc_d/(1-\gamma)G_d(s)$, which holds for $c_d > 0$ small. In particular, $\sigma_1 > rc_d/(1-\gamma)G_d(s)$ whenever $c_d \leq c_M(\gamma)$, in which case $\underline{\gamma} < \gamma < \bar{\gamma}$ is sufficient for existence. Hence, if $c_d \leq c_M(\gamma)$ and $\underline{\gamma} \leq \gamma \leq \bar{\gamma}$ then $\sigma = \sigma_1$, $\delta = 1$ and $\omega = 1$. In this case $P_s = P_s^* = \sigma_1$ and $\omega = 1$ with $P_d + P_s = 1$.

Finally, all equilibria coexist when $c_d \leq c_L(\gamma)$ and $\underline{\gamma} \leq \gamma \leq \bar{\gamma}$. To see why, note from Lemma

A, that $\bar{c} > c_M(\gamma) > c_L(\gamma)$ when $\underline{\gamma} \leq \gamma \leq \bar{\gamma}$. Hence, if $c_d \leq c_L(\gamma)$, then $\underline{\gamma} \leq \gamma \leq \bar{\gamma}$ is sufficient for existence of all possible equilibria.■

Proof of Corollary 3. Start by recognizing that if $c_d \leq \bar{c}$, then $G_s(s) \geq \phi + rc_d$. Hence, $rW(1,0) = u_n + G_s(s) - \phi - rc_d > rW(0,0) = rV_n = u_n$. Now consider the remaining equilibria where $P_s = P_s^*$ and either (i) $P_d = 0$, (ii) $0 < P_d < 1 - P_s^*$, or (iii) $P_d = 1 - P_s^*$. In the first two cases $\delta = \delta^*$. Since $\sigma = \sigma^*$, then we have $V_n = V_d - c_d$ and $V_s - c_s = V_n + c_d$. Hence, $rW(P_s, P_d) = rV_n = u_n$. In case (iii) we have $\delta = 1$ so that $rW(P_s^*, P_d^*) = u_n + (1-\gamma)P_s^*\omega G_d(s) - rc_d$. Clearly, in this case $rW(P_s^*, P_d^*) < rW(1,0)$ because $P_s + P_d = 1$ in both cases, but $P_s^* < 1$. Also, $rW(P_s^*, P_d^*) > u_n$ because $V_s - c_s - c_d = V_d - c_d > V_n$.■

Proof of Corollary 4. To prove that students tend to underinvest in skill when education is cheap, notice from (13) that $(P_s, P_d) = (P_s^*, P_d^*)$ requires $c_d \leq c_M(\gamma)$. When c_d rises above $c_M(\gamma)$, we always have $P_d = 0$, which means that if $P_s > 0$ then all students invest in skill.

To demonstrate that students tend to underinvest in skill when $G_s(s)/G_d(s)$ is small, note that $\sigma = \sigma^*$ (under-investment in skill) is possible only if $\gamma \leq \bar{\gamma}$. Notice that $\sigma = 1$ is possible only if $\gamma \geq \underline{\gamma}$. Finally, observe that $\bar{\gamma} = \underline{\gamma} = 1$ if $G_s(s)/G_d(s) = 0$, while $\underline{\gamma}$ and $\bar{\gamma}$ decrease in $G_s(s)/G_d(s)$. In particular, if $G_s(s) \geq G_d(s)$ then $\underline{\gamma} = 0$, while $\lim_{G_s(s)/G_d(s)\to\infty} \bar{\gamma} = 0$. Thus, as $G_s(s)/G_d(s)$ increases the parameters space $(0, \bar{\gamma})$ that sustains equilibria with $\sigma = \sigma^*$ shrinks, while the parameter space $[\underline{\gamma}, 1)$ that sustains equilibria with $\sigma = 1$, increases.

We now prove that the smallest steady state skill level is associated with economies with low costs of education. To do so consider all equilibria where $P_s > 0$. First, consider the cases where $\delta = 1$ hence $P_s + P_d = 1$. Three such equilibria exist: (i) $P_s = 1$ if $c_d \leq \bar{c}$, (ii) $P_s = P_{M,1} = \sigma_1$ if $c_d \leq c_M(\gamma)$, and (iii) $P_s = P_{L,1} = \sigma_\omega$ if $c_d \leq c_L(\gamma)$. Note that σ_1 and σ_ω are independent of c_d. Second, consider equilibria where $\delta = \delta^*$ so that $P_s + P_d < 1$. Three such equilibria exist: (i) $P_s = P_H = \frac{rc_d}{G_s(s)}$ if $c_d \leq \bar{c}$, (ii) $P_s = P_{M,2} = \frac{rc_d}{(1-\gamma)G_d(s)}$ if $c_d \leq c_M(\gamma)$, and (iii) $P_s = P_{L,2} = \frac{rc_d}{\omega^*(1-\gamma)G_d(s)}$ if $c_d \leq c_L(\gamma)$. Letting $j = L, M$, note that $P_{j,1}$ and $P_{j,2}$ coexist, that $P_{j,1} > P_{j,2}$, and that P_H and $P_{j,2}$ fall as c_d falls. Hence, the smallest steady state skill level P_s is achieved when $c_d \leq c_L(\gamma)$.■

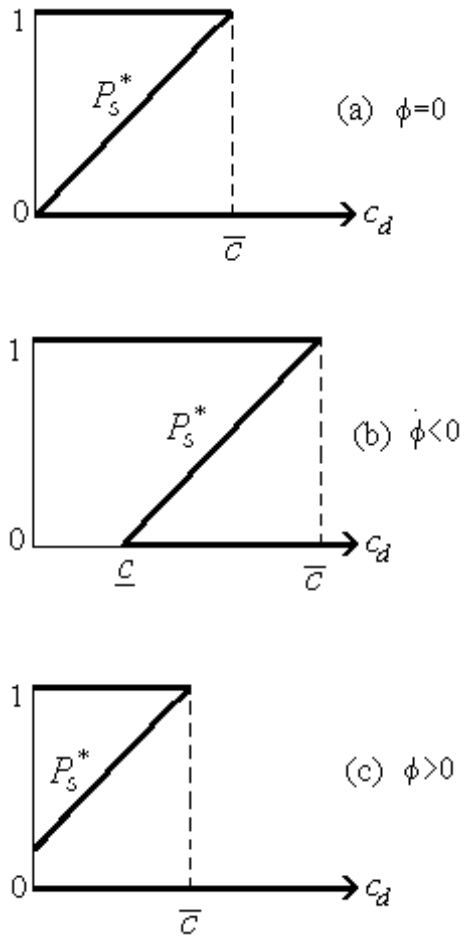
1
P_s^*
(a) ϕ=0
0
c_d
$\overline{c}$
1
P_s^*
(b) $\dot{\phi}$<0
0
c_d
$\underline{c}$
$\overline{c}$
1
P_s^*
(c) ϕ>0
0
c_d
$\overline{c}$

Figure 1

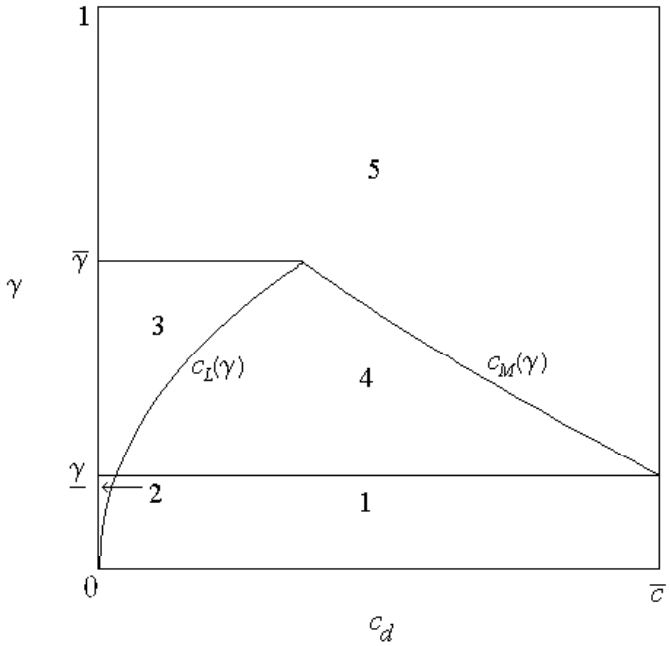

Figure 2

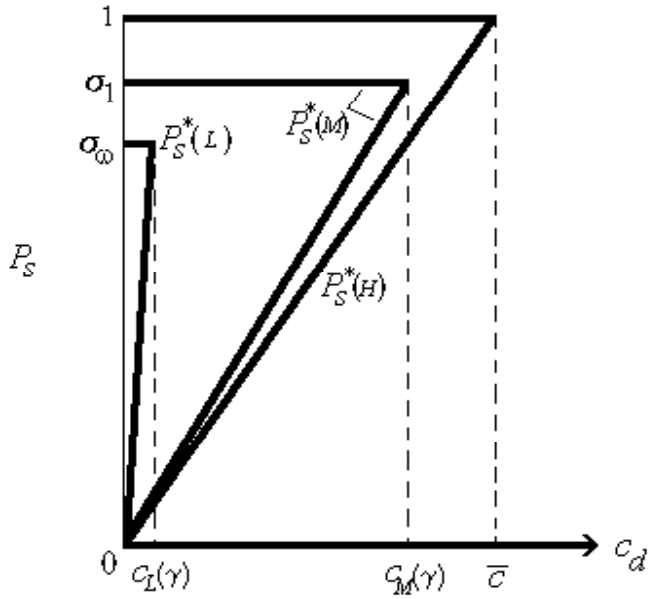

Figure 3