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Characterising Competitive Equilibrium in Terms of Opportunity

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Introductory note

This paper is the first draft of a technical appendix to a chapter of a book I am writing, with the provisional title *The Community of Advantage*. The central argument of the book will be that many elements of the (classically) liberal tradition of normative economics do not depend on assumptions about individual rationality, and so it is possible for a behavioural economist to work in that tradition. I will propose an approach to normative economics that differs both from neoclassical welfare economics and from the various variants of soft paternalism that are currently being proposed by behavioural economists. My approach has two distinctive features. First, it is written from a contractarian perspective. That is, it is addressed to citizens as potential parties to mutually beneficial agreements, and not to an imagined benevolent despot or social planner. (A first draft of this part of the argument has been published as Sugden [2013].) Second, its normative criterion is opportunity, not welfare, happiness or well-being.

Sections 1 to 4 of this paper follow the analysis in McQuillin and Sugden (2012), specialised to the one-period case and with minor changes in notation. The set-up, and the definition of the 'opportunity criterion' are slightly different from those used in Sugden (2004). The differences are explained in McQuillin and Sugden (2012). The argument in Section 5 is new.

1 The Opportunity Criterion

An *exchange economy* is defined by a set of two or more *individuals* $i = 1, ..., N$, a set of two or more infinitely-divisible *goods* $g = 1, ..., G$, and for each individual i, a non-negative *endowment* $e_{i,g}$ of *claims* on each good g, such that for each good, the total of all individuals' endowments is strictly positive. The only relevant activity is the 'acquisition' and 'disbursement' of claims by individuals, which takes place in a single period.

A claim on a unit of good g confers on its holder both an entitlement and an obligation to consume one unit of that good at the end of the period. There is no general option of free disposal (hence the 'obligation' to consume). 'Consumption' need not be interpreted as something that individuals value positively; it represents whatever opportunities and obligations an individual incurs by virtue of holding a claim at the end of the period. For example, g might be an obsolete type of television; 'consumption' might take the form of unwanted storage or costly disposal. However, good 1 (*money*) will be interpreted as a good whose consumption is always valued positively. This property of money has to be treated as a matter of interpretation, because there is no formal concept of preference in the model. Money serves as the medium of exchange and as the standard of value.

In interpreting the model, it is useful to imagine that economic activity is organised by some *trading agency*, distinct from the 'individuals' of the economy. This agency might be thought of as an 'auctioneer' in the sense of Walrasian general equilibrium theory, or as a 'social planner' in the sense of modern welfare economics, or as a set of competing profit-seeking 'traders' who come to the economy from outside (as in the model of Sugden, 2004).[1] The trading agency offers a set of trading opportunities to each individual.

For a given exchange economy, individuals' opportunities are defined in terms of *net acquisition*. For each individual i, for each good g, net acquisition of g by i is denoted $\Delta_{i,g}$. This is to be interpreted as the additional claims on good g taken on by individual i during the period, minus any claims disbursed. Each $\Delta_{i,g}$ is required to be a real number in the interval $[-e_{i,g}, \infty)$. Since $e_{i,g} + \Delta_{i,g}$ represents i's consumption of g, this requirement rules out negative

[1] It would be possible to close the model by assuming an auctioneer or social planner who is a government employee, any positive or negative surplus from whose trading operations accrues to individuals as lump-sum benefits or taxes. Analogously, one might assume a set of traders employed by firms whose shares are entirely owned by individuals. However, the model is simpler and more transparent if the trading agency is entirely separate from the individuals.

consumption. A vector $\Delta_i = (\Delta_{i,1}, \ldots, \Delta_{i,G})$ of net acquisitions is a *behaviour* by i. An *opportunity set* for an individual i, denoted O_i, is a set of behaviours for that individual; the interpretation is that i must choose one element from O_i. A behaviour Δ_i is *allowable* in O_i if and only if Δ_i is an element of that set. A profile $O = (O_1, \ldots, O_N)$ of opportunity sets is a *regime*. A behaviour profile $\Delta = (\Delta_1, \ldots, \Delta_N)$ is allowable in regime O if and only if each Δ_i is allowable with respect to O_i. The set of behaviour profiles that are allowable in regime O (i.e. the Cartesian product $O_1 \times \ldots \times O_N$) is denoted $A(O)$. A regime can be interpreted as a specification of the opportunities made available to individuals by the trading agency. My object is to assess alternative regimes for a given economy.

A behaviour profile Δ is *feasible* if and only if, for each good g, $\sum_i \Delta_{i,g} = 0$. These feasibility constraints represent the resource limitations of the economy, under the assumption that all goods are initially held by individuals as endowments; they are strict equalities because there is no free disposal option. Notice that a behaviour profile can be allowable even if it is infeasible. This allows the model to represent a state of affairs that is common in all real-world economies: for each individual, the elements of her opportunity set appear unconditionally feasible *for her*, but the Cartesian product of those sets may contain elements that are not feasible *for the economy as a whole*. (For example, every individual separately may have the opportunity to buy a certain good at a certain price, but if they all tried to exercise this opportunity simultaneously, their demands could not be met.)

For each regime O, I assume that the behaviour of each individual is uniquely determined. The *chosen behaviour* of individual i in regime O is denoted by $\Delta_i(O)$; the profile $(\Delta_1[O], \ldots, \Delta_N[O])$ of chosen behaviour for all individuals is denoted by $\Delta(O)$. In general, chosen behaviour can be feasible or infeasible. Notice that no assumptions are being made about the mechanism that determines what each individual chooses from her opportunity set. Choices may be rational or irrational: all that is being assumed is that, from the viewpoint of the modeller, they are predictable.

For any individual i and any behaviours Δ_i and Δ'_i, Δ'_i *dominates* Δ_i if and only if (i) $\Delta'_{i,1} > \Delta_{i,1}$ and (ii) for each $g \geq 2$, $\Delta'_{i,g} = \Delta_{i,g}$. Given the implicit assumption that consumption of money is always valued positively, a dominated behaviour Δ_i is unambiguously less desirable than the behaviour Δ'_i that dominates it. I will say that a behaviour Δ_i is *dominated in* an opportunity set O_i if and only if there is some behaviour $\Delta'_i \in O_i$ such that Δ'_i

dominates Δ_i, and that a behaviour profile Δ is *dominated in* regime *O for individual i* if and only if Δ_i is dominated in O_i.

I now define a criterion against which, for a given economy, any regime can be assessed:

> *Opportunity Criterion*. A regime O satisfies the Opportunity Criterion if $\Delta(O)$ is feasible, and if, for every feasible behaviour profile Δ', *either* Δ' is allowable in O *or* there is some individual i such that Δ'_i is dominated in O_i.

To understand the normative intuition behind the Opportunity Criterion, consider a regime O for which the chosen behaviour profile $\Delta(O)$ is feasible. Thus, the opportunities specified by $A(O)$ can be made available to individuals without any breach of feasibility constraints. If, despite this, the Opportunity Criterion is not satisfied, there is an additional putative opportunity, namely the behaviour profile Δ', that is feasible and that is non-dominated in O for every individual, but that has *not* been made available. Since, for each individual i, Δ'_i is non-dominated in O_i, no argument about dominance can be deployed to show that, had that opportunity been made available in addition to those given by O, some individual would not have wanted to take it up. The implication is that individuals collectively lack the opportunity to make a combination of choices that they might all want to make and that is compatible with the resource constraints of the economy. The Opportunity Criterion requires that individuals are not deprived in this way.

2 The Market Opportunity Theorem

I now characterise a particular type of regime for an exchange economy – a *single-price regime*. In such a regime, for each non-money good $g = 2, \ldots, g$, there is a *market price* p_g expressed in money units; this price is finite, and may be positive, zero or negative. As a matter of notation, it is convenient to represent the idea that money is the medium of exchange by defining $p_1 = 1$. Each individual is free to keep her endowments if she chooses, but also to exchange claims on non-money goods for claims on money (and vice versa) on terms that are at least as favourable as those implied by market prices, subject to the constraint that her holdings of claims on any good cannot be negative. More formally:

> *Single-price regime*. A regime O is a single-price regime if there exists a finite, real-valued price vector $p = (p_1, \ldots, p_G)$ such that $p_1 = 1$ and, for each individual i,

every behaviour Δ_i that satisfies $\Sigma_g\, p_g\, \Delta_{i,g} = 0$ is either allowable or dominated in O_i.

A single-price regime O is *market-clearing* if the chosen behaviour $\Delta(O)$ is feasible. Later, I will show that, given certain weak assumptions, a market-clearing single-price regime exists for every exchange economy. For the moment, however, I simply examine the properties of such regimes. The reader may be surprised that my definition of a single-price regime allows the possibility that individuals are free to trade on *more* favourable terms than those implied by market prices. This may seem an unnecessary complication, but it will prove to be useful later.

The following theorem identifies one important property of market-clearing single-price regimes:

Market Opportunity Theorem. For every exchange economy, every market-clearing single-price regime satisfies the Opportunity Criterion.

This is a special case of a more general result that is proved by McQuillin and Sugden (2012).

A full proof of the Market Opportunity Theorem is given in the Appendix, but the idea behind the proof can be seen easily in the case of an exchange economy with just two individuals and two goods. Such an economy can be described by an Edgeworth box diagram like that in Figure 1. In this diagram, consumption of good 1 by the two individuals is measured on the horizontal axis, consumption of good 2 on the vertical. The origin for individual 1 is the bottom left corner of the box, with positive consumption above and to the right; the origin for individual 2 is the top right of the box, with positive consumption below and to the left. Endowments are shown by the point E. Every point in the box (and no other) describes a profile of consumption that can be reached by a feasible behaviour profile. It must be remembered that the diagram describes individuals' *holdings* of goods, rather than net acquisitions. However, if endowments are treated as given, there is a simple one-to-one relationship between net acquisitions and points in the diagram, interpreted holdings of goods at the end of the trading period (or, equivalently, as consumption): for each individual i, for each good g, i's final holding of g is $e_{i,g} + \Delta_{i,g}$. I will refer to properties of the diagram as 'representing' properties of net acquisitions.

Consider a market-clearing single-price regime, defined by a particular price p_2 for good 2. The case shown in Figure 1 has $p_2 > 0$, but this is not essential for the argument. The downward-sloping *price line* through E should be interpreted as having a gradient of $-1/p_2$.

Assume that each individual is allowed to trade *only* at this price. Then the opportunity set O_1 is represented by the set of points on the line AA′ (i.e. the solid and dotted line segments of the price line). Notice that this set can include behaviours for individual 1 that are outside the Edgeworth box and so cannot be realised in any regime, given the economy's resource constraints. Points that are in the box and to the left of AA′ represent behaviours that are dominated for individual 1. Similarly, O_2 is represented by the set of points on the line BB′ (i.e. the solid and dashed segments of the price line); points that are in the box and to the right of BB′ represent behaviours that are dominated for individual 2. Since the regime is market-clearing, the chosen behaviour profile is feasible and can be represented by a single point C on AA′ and BB′.

The set of feasible behaviour profiles is represented by the set F of all points in the Edgeworth box. Consider the following subsets of F: F_0 is the set of points in F that represent allowable behaviour profiles; F_1 is the set of points in F that represent behaviours that are dominated for individual 1; and F_2 is the set of points in F that represent behaviours that are dominated for individual 2. It follows immediately from the definition of the Opportunity Criterion that if the chosen behaviour profile is feasible (as it is in this case) and if every point in the box is in at least one of the three subsets, that criterion is satisfied. In this case, F_0 is the set of points on the line BA′ (i.e. the intersection of the price line and the box); F_1 is the set of points that are in the box and to the left of BA′; and F_2 is the set of points that are in the box and to the right of BA′. So every point in the box is in one (and in fact only one) of the three subsets: the Opportunity Criterion is satisfied.

So far, in discussing this example, I have assumed that individuals can trade *only* at the price p_2. However, the definition of a single-price regime requires only that, for each individual, every behaviour that can be described as trading at that price is allowable *or dominated*. In terms of the diagram, this implies that for any point in the Edgeworth box: if it is to the left of BA′, it is in F_1; if it is to the right of BA′, it is in F_2; and if it is on BA′, it is in at least one of the sets F_1, F_2 or F_3. And so a single-price market-clearing regime must satisfy the Opportunity Criterion.

3 The First Fundamental Theorem of welfare economics

The Market Opportunity Theorem is closely related to the First Fundamental Theorem of welfare economics, which states that every competitive equilibrium is Pareto-efficient. The crucial difference is that the First Fundamental Theorem is about *preference-satisfaction*, and presupposes that individuals have coherent preferences to be satisfied, while the Market Opportunity Theorem is about *opportunity*, and requires no such presupposition. In this Section, I investigate the relationship between the two theorems, in the context of a given exchange economy.

As a first step, I define a formal concept of preference. Preference is usually defined over consumption bundles but, if endowments are treated as fixed, it can equivalently be defined over behaviours. For any individual i, *weak preference* is a binary relation $\geq_i$ on the set of conceivable behaviours for individual i; for any behaviours Δ_i and Δ'_i, $\Delta_i \geq_i \Delta'_i$ is read as 'i weakly prefers Δ_i to Δ'_i'. Strict preference for Δ_i over Δ'_i, denoted by $\Delta_i >_i \Delta'_i$, is defined to be equivalent to ($\Delta_i \geq_i \Delta'_i$ and *not* $\Delta'_i \geq_i \Delta_i$). Indifference between Δ_i and Δ'_i, denoted by $\Delta_i \sim_i \Delta'_i$, is defined to be equivalent to ($\Delta_i \geq_i \Delta'_i$ and $\Delta'_i \geq_i \Delta_i$). The weak preference relation is *complete* if, for all behaviours Δ_i and Δ'_i, *either* $\Delta_i \geq_i \Delta'_i$ *or* $\Delta'_i \geq_i \Delta_i$ (or both). It is *reflexive* if, for all behaviours Δ_i, $\Delta_i \geq_i \Delta_i$. It is *transitive* if, for all behaviours Δ_i , Δ'_i and Δ_i'', [$\Delta_i \geq_i \Delta'_i$ and $\Delta'_i \geq_i \Delta''_i$] implies $\Delta_i \geq_i \Delta''_i$. It is an *ordering* if it is complete, reflexive and transitive. It *respects dominance* if, for all Δ_i, Δ'_i, [Δ'_i dominates Δ_i] implies $\Delta'_i >_i \Delta_i$. I will say that, for a given individual i and weak preference relation $\geq_i$, i's choices are *rationalised by* $\geq_i$ if, in every regime O, i's chosen behaviour $\Delta_i(O)$ is weakly preferred to every element of O_i. In neoclassical welfare economics, it is conventional to use assumptions which, translated into the present theoretical framework, have the following implication:

> *Existence of Preferences*. For each individual i there is a preference ordering $\geq_i$ that respects dominance and that rationalises i's choices.

Assuming Existence of Preferences, I now define Pareto-efficiency. For any regime O and any feasible behaviour profiles Δ and Δ', Δ is *weakly Pareto-preferred* to Δ' if, for every individual i, $\Delta_i \geq_i \Delta'_i$; this Pareto preference is *strict* if in addition there is some individual j for whom $\Delta_j >_j \Delta'_j$. A feasible behaviour profile Δ is *Pareto-efficient* if it is weakly Pareto-preferred to every other feasible behaviour profile. The Market Opportunity Theorem and the First Fundamental Theorem are connected by the following result:

> *Linkage Result*. For every exchange economy and for every regime O of that economy, if the chosen behaviour profile $\Delta(O)$ satisfies the Opportunity Criterion and if Existence of Preferences holds, then $\Delta(O)$ is Pareto-efficient.

It follows immediately from the Market Opportunity Theorem and the Linkage Result that in every market-clearing single-price regime, if Existence of Preferences holds, the chosen behaviour profile is Pareto-efficient. And that, in the context of an exchange economy, is the First Fundamental Theorem.

4 The Strong Opportunity Criterion

To say that the Opportunity Criterion is satisfied is to say that individuals collectively are not deprived of opportunities to make combinations of choices that are feasible and non-dominated. But notice that that criterion is framed in terms of *behaviour profiles*, and a behaviour profile describes the net acquisitions of *every* individual in the economy. Thus, the phrase 'individuals collectively' means '*all* individuals in the economy, considered together'. One might think that the Opportunity Criterion fails to take account of the presence or absence of opportunities for feasible combinations of choices by sets of individuals that do not contain everyone.

Consider the economy shown in Figure 2, which is drawn using the same conventions as Figure 1. Each individual's opportunity set contains *only* those behaviours represented by points on the price line. In this economy, there is a single price p_2 at which both individuals are free to exchange the two goods (such that the gradient of the line through C is $-1/p_2$), and the chosen behaviour profile (represented by C) is feasible. However, neither player's opportunity set includes the behaviour **0** that corresponds with keeping one's endowments, and for individual 1, this behaviour is non-dominated. One might think of this regime as one in which there is a compulsory transfer of goods from individual 1 to individual 2 before trade is allowed; after this transfer, both individuals are allowed to trade freely at the price p_2, and the market clears. This regime satisfies the Opportunity Criterion. The proof that it does so is exactly the same as in the case of the regime in Figure 1: feasible points on the price line are allowable for both individuals; feasible points to the left of the line are dominated for individual 1; and feasible points to the right of it are dominated for individual 2. Nevertheless, there is a behaviour for individual 1, namely **0**, that uses only the resources with which individual 1 is endowed, but that is neither allowable nor dominated in O_1. In other words, individual 1 is deprived of the opportunity to choose something that *he* might want to choose, and that is feasible within the resource constraints imposed by *his own* endowments. If one thinks of individuals as having entitlements to their endowments, the absence of this opportunity for individual 1 is analogous with the absences of opportunity that contravene the Opportunity Criterion.

To allow a general analysis of opportunities for sets of individuals that do not contain everyone, let I be the set of all N individuals in the economy, and let $S \subseteq I$ be any set of one or more of those individuals. For any regime O, the *opportunity profile for S*, denoted by O_S, is a list of the opportunity sets O_i for all individuals i in S. A *behaviour profile for S*, denoted by Δ_S, is a list of behaviours Δ_i, one for each individual i in S. Such a profile is *allowable in* O_S if each of its component behaviours Δ_i is allowable in O_i; the set of profiles that are so allowable is $A_S(O)$. It is *feasible for S* if, for each good g, the sum of net acquisitions of g by all members of S is zero.

The following criterion strengthens the Opportunity Criterion to assess opportunities for all sets of individuals:

> *Strong Opportunity Criterion*. A regime O satisfies the Strong Opportunity Criterion if $\Delta(O)$ is feasible, and if, for every non-empty set of individuals $S \subseteq I$ and for every behaviour profile Δ'_S that is feasible for S, *either* Δ'_S is allowable in O_S *or* there is some individual $i \in S$ such that Δ'_i is dominated in O_i.

In relation to the set $S = I$, this criterion is the same as the Opportunity Criterion; so any regime that satisfies the Strong Opportunity Criterion also satisfies the Opportunity Criterion. But the Strong Opportunity Criterion imposes requirements analogous to those of the Opportunity Criterion for *every* set of individuals. It requires, for each such set S, that the members of S are not deprived of opportunities to make combinations of choices that they might want to make and that are feasible within the resource constraints imposed by their combined endowments. In relation to each singleton set $S = \{i\}$, it requires that the behaviour $\Delta_i = \mathbf{0}$ is allowable in O_i (and requires nothing else). That is, it requires that each individual i has the opportunity to consume exactly what he was endowed with.

The Market Opportunity Theorem can be strengthened in a similar way:

> *Strong Market Opportunity Theorem*. For every exchange economy, every market-clearing single-price regime satisfies the Strong Opportunity Criterion.

5 Characterising market-clearing single-price regimes

So, for a regime to satisfy the Strong Opportunity Criterion, it is *sufficient* that that regime is single-price and market-clearing. It is natural to ask whether this condition is also *necessary*.

It turns out the answer is 'No'. Consider the case shown in Figure 3. This diagram, which uses the same conventions as Figures 1 and 2, shows a regime O for a two-person, two-good economy in which both individuals have non-zero endowments of both goods, and there are two different (strictly positive) prices at which good 2 can be traded – a high price (call this p_2^H), and a low price (p_2^L). Individual 1 is allowed to buy good 2 at the low price and sell at the high price, while the opposite is true of individual 2. The market clears at the high price, with individual 2 buying from individual 1. Since every point in the Edgeworth box is either allowable to both individuals or dominated for one of them, the Opportunity Criterion is satisfied. Since there are only two individuals, the only additional requirement of the *Strong* Opportunity Criterion is that the behaviour **0** is allowable for each individual, and this requirement is satisfied too.

It is probably obvious that O is not a single-price regime, but it will be useful to show formally that it is not. For each individual $i = 1, 2$, let Ω_i be the set of behaviours Δ_i that are neither allowable nor dominated in O_i. In Figure 3, Ω_1 is represented by the set of points above and to the right of AEA′. Similarly, Ω_2 is represented by the set of points below and to the left of AEA′. Figure 4 plots Ω_1 and Ω_2 using a different coordinate system. In this diagram, the horizontal axes measures net acquisition (by either individual) of good 1, and the vertical axis measures net acquisition of good 2. Ω_1 is the set of points above and to the right of the *dotted* frontier; Ω_2 is the set of points above and to the right of the *dashed* frontier. Now (as the first step in a proof by contradiction) suppose that O is a single-price regime. By the definition of such a regime, there exists a finite price p_2 such that every behaviour Δ_2 that satisfies $\Delta_{2,1} + p_2\Delta_{2,2} = 0$ is either allowable or dominated in O_2. This is equivalent to saying that the line through **0** with gradient $-1/p_2$ does not pass through Ω_2. But it is immediately obvious from the diagram that, whatever the value of p_2, this line *does* pass through Ω_2. So the supposition that O is a single-price regime is false.

However, there is a sense in which, for a large enough economy, any regime that satisfies the Strong Opportunity Criterion is 'almost' a single-price regime. The concept of 'largeness' that I will use derives from Francis Ysidro Edgeworh (1881). The intuitive idea is to take some model economy, replicate every component of it, and then create a larger economy by combining the original economy and its replica. By adding more and more replicas, one can create larger and larger economies which are identical to one another except for scale. The beauty of this method is that it allows one to investigate the effect of changing the scale of an economy while holding other features constant.

As an illustration, consider the two-person exchange economy described by Figures 3 and 4; I will call this economy X^1. Now consider the effect of combining this economy with one exact replica. This creates a four-person economy X^2 in which individuals 1 and 3 (the *odds*) have the same endowments as individual 1 in X^1, and individuals 2 and 4 (the *evens*) have the same endowments as individual 2 in X^1. The regime shown in Figures 3 and 4 (formerly called O) will now be called O^1, to signify that it is a regime for economy X^1. This regime can be replicated to create a regime O^2 for economy X^2. Let $O^r{}_i$ denote the opportunity set of individual i in regime O^r (r = 1, 2), and let $\Delta^r{}_i\,(O^r)$ denote the chosen behaviour of individual i that regime. Since opportunity sets have been replicated, $O^1{}_1 = O^2{}_1 = O^2{}_3$ and $O^1{}_2 = O^2{}_2 = O^2{}_4$. I assume that chosen behaviours are replicated too, so that $\Delta^1{}_1(O^1) = \Delta^2{}_1(O^2) = \Delta^2{}_3(O^2)$ and $\Delta^1{}_2(O^1) = \Delta^2{}_2(O^2) = \Delta^2{}_4(O^2)$. Since, by assumption, O^1 is market-clearing, O^2 is market-clearing too. However, although O^1 satisfies the Strong Opportunity Criterion, O^2 does not.

This is because, in the four-person economy, the two even individuals are deprived of opportunities for trade *between themselves* that are feasible *for them*. Since the present paragraph refers only to this economy, I simplify the notation by suppressing the '2' superscripts that identify it. Consider any price p_2 in the interval $p_2{}^{\mathrm{H}} > p_2 > p_2{}^{\mathrm{L}}$. Let x be any quantity of good 2 such that $e_{2,2} = e_{4,2} \geq x \geq 0$ and $e_{2,1} = e_{4,2} \geq p_2x$. Define $\Delta_2 = (-p_2x, x)$ and $\Delta_4 = (p_2x, -x)$. Thus, (Δ_2, Δ_4) is a behaviour profile for the set {2, 4} of even individuals, where individual 2 buys x units of good 2 at price p_2, and individual 4 sells x units at the same price. Clearly, this behaviour profile is feasible for {2, 4}. But, as Figure 4 shows, Δ_2 is not in Ω_2 and (since Ω_4 is identical to Ω_2) Δ_4 is not in Ω_4. In other words, Δ_2 is neither allowable nor dominated in O_2, and Δ_4 is neither allowable nor dominated in O_4. So the Strong Opportunity Criterion is not satisfied.

The intuitive idea is that the larger the scale of an economy, the more difficult it is to find a market-clearing regime that satisfies the Strong Opportunity Criterion but is not single-price. I now present two general results that formalise that idea. The first result establishes that as the scale of an economy increases, the set of regimes that satisfy the Strong Opportunity Criterion 'shrinks':

Shrinkage Theorem. Let (X^1, O^1) be any pair of an exchange economy and a regime for that economy. For any integer $r > 0$, let (X^r, O^r) be the economy and regime created by combining (X^1, O^1) with r–1 replicas. Then if O^r fails to satisfy the Strong Opportunity Criterion, so too does O^{r+1}.

The second result shows that, in the limit as the scale of an economy increases indefinitely, the only regimes that satisfy the Strong Opportunity Criterion are those that are 'almost the same as' market-clearing single-price regimes. To present this result, I need some additional definitions. Consider any exchange economy X and any regime O for that economy. For any individual i, for any behaviour Δ_i and any finite real number $\varepsilon > 0$, let $\nu(\Delta_i, \varepsilon)$ be the set of behaviours whose (Euclidian) distance from Δ_i is no greater than ε. I will say that Δ_i is 'within ε of being allowable', or ε-*allowable*, in O_i if there is some behaviour $\Delta'_i \in \nu(\Delta_i, \varepsilon)$ that is allowable in O_i. I will say that Δ_i is ε-*dominated* in O_i if there is some behaviour $\Delta'_i \in \nu(\Delta_i, \varepsilon)$ that is dominated in O_i. And I will say that O is an ε-*single-price regime* if there exists a finite, real-valued price vector $p = (p_1, \ldots, p_G)$ such that, for each individual i, every behaviour Δ_i that satisfies $\Sigma_g\, p_g\, \Delta_{i,g} = 0$ is either ε-allowable in O_i or ε-dominated in O_i. Thus, at sufficiently small values of ε, ε-single-price regimes are 'almost the same as' single-price regimes. The second result can now be stated as:

Convergence Theorem. Let (X^1, O^1) be any pair of an exchange economy and a regime for that economy. For any integer $r > 0$, let (X^r, O^r) be the economy and regime created by combining (X^1, O^1) with r–1 replicas. For any given $\varepsilon > 0$, if O^r satisfies the Strong Opportunity Criterion for all $r > 0$, O^r is an ε-single-price regime.

6 Existence of a market-clearing single-price regime

The idea of a single-price market-clearing regime is an equilibrium concept. In such a regime, each non-money good has a price; given individuals' endowments, these prices define individuals' opportunity sets; given those opportunity sets, individuals choose their behaviours; and those behaviours are such that all markets clear. If prices were fixed arbitrarily, there would be no general reason to expect markets to clear. The implicit assumption is that the trading agency *sets* prices that clear markets. But is there any guarantee that a market-clearing equilibrium exists? Proofs of the existence of 'competitive' equilibrium typically assume that individuals act on coherent preferences, but the conceptual framework that I am using does not allow this. In this Section, I present an existence theorem that does not require strong assumptions about preferences.

Consider any exchange economy. A *price vector* for that economy is a finite, real-valued vector $p = (p_1, \ldots, p_G)$ with $p_1 = 1$. A regime O is a *strict single-price regime* if there is some price vector p such that, for each individual i and each behaviour Δ_i, Δ_i is allowable *if and only if* $\Sigma_g\, p_g\, \Delta_{i,g} = 0$. (Notice that if O is a strict single-price regime, there is one and only one such p.) I have already assumed that, for any given regime O, there is a chosen behaviour $\Delta_i(O)$ for each i. Since any strict single-price regime is fully described by its price vector, we can express the chosen behaviour Δ_i of each individual i as a function of p. Thus, for each good $g = 1, \ldots, m$, we can write *net excess demand* for g, i.e. the sum of the chosen values of $\Delta_{i,g}$ for all individuals i, as a function $x_g(p)$. O is *market-clearing* if $x_g(p) = 0$ for every good g. Notice that if $x_g(p) = 0$ holds for all *non-money* goods $g = 2, \ldots, G$, it necessarily holds for good 1 too. More generally, *the value of* net excess demand, expressed in money units by using the price vector p and summed over all individuals and all goods (including money), is identically equal to zero, irrespective of whether markets clear. That is, $\Sigma_i\, \Sigma_g\, p_g\, x_g(p) = 0$. This identity (a version of *Walras's Law*) is an implication of the assumption that each individual's chosen behaviour is in his opportunity set; it does not depend on any assumptions about preferences.

Now consider the following two additional assumptions:

Continuity. For each non-money good $g = 2, \ldots, G$, $x_g(p)$ is a continuous function.

Intrinsic Value of Money. For each non-money good $g = 2, \ldots, G$, there is an *upper limit* price $p_g^{\mathrm{U}} > 0$ and a *lower limit* price $p_g^{\mathrm{L}} < 0$, such that, for all price vectors p, $p_g \geq p_g^{\mathrm{H}}$ implies $x_1(p) > 0$, and $p_g \leq p_g^{\mathrm{L}}$ implies $x_1(p) > 0$.

Since I am not assuming that individuals act on coherent preferences, I cannot follow the neoclassical strategy of deriving Continuity as a property of the demand functions of rational individuals whose preferences are 'well-behaved'. However, I suggest that Continuity is a plausible assumption about *aggregate* behaviour in a large economy. Intrinsic Value of Money expresses the idea that money is always perceived as a desirable consumption good, and that this desire is never satiated. Intuitively, if the price of some non-money good g is sufficiently high, individuals who have positive endowments of g will want to take advantage of the opportunity to acquire large amounts of money by giving up small amounts of g, and so money will be in excess demand. Similarly, if the price of some non-money good (or, in this case, bad) *g* is sufficiently negative, individuals will want to take advantage of the opportunity to acquire large amounts of money by taking on small amounts of *g*, and so again money will be in excess demand.

The following theorem can be proved:

> *Existence Theorem*. For any exchange economy, if Continuity and Instrinsic Value of Money are satisfied, there exists a single-price market-clearing regime.

Figure 1: A market-clearing single-price regime

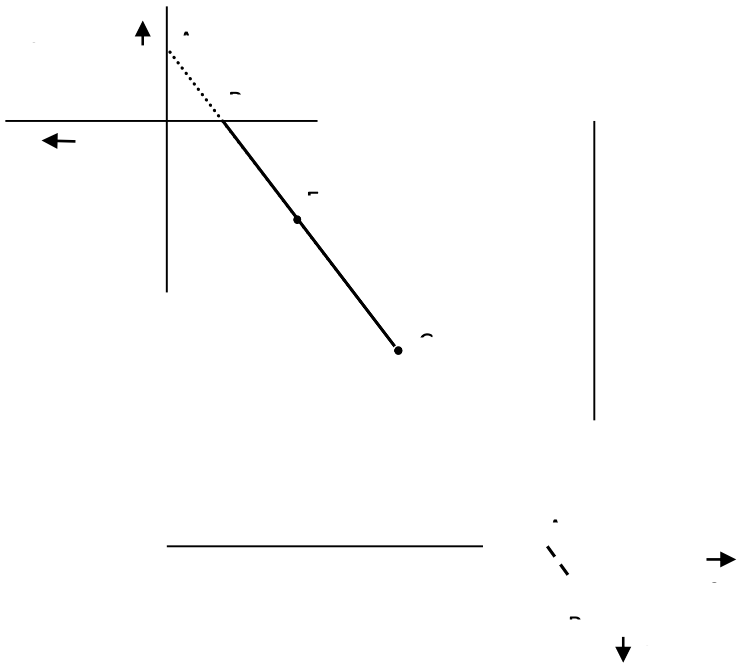


Figure 2 A regime that satisfies the Opportunity Criterion but does not respect endowments

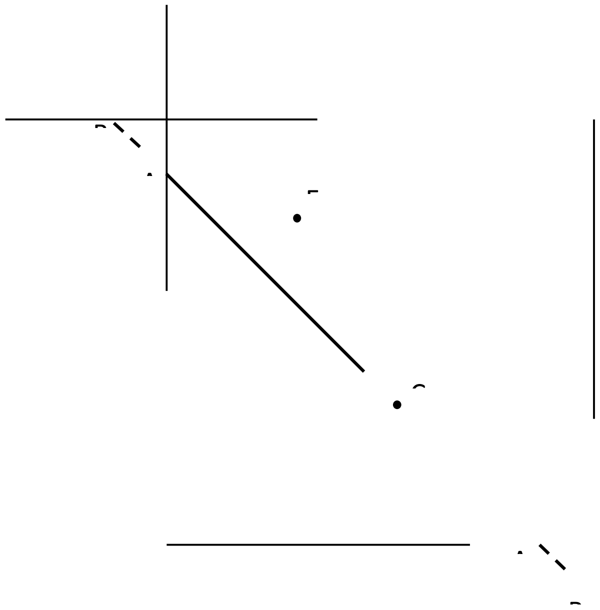


Figure 3 A non-single-price regime that satisfies the Strong Opportunity Criterion

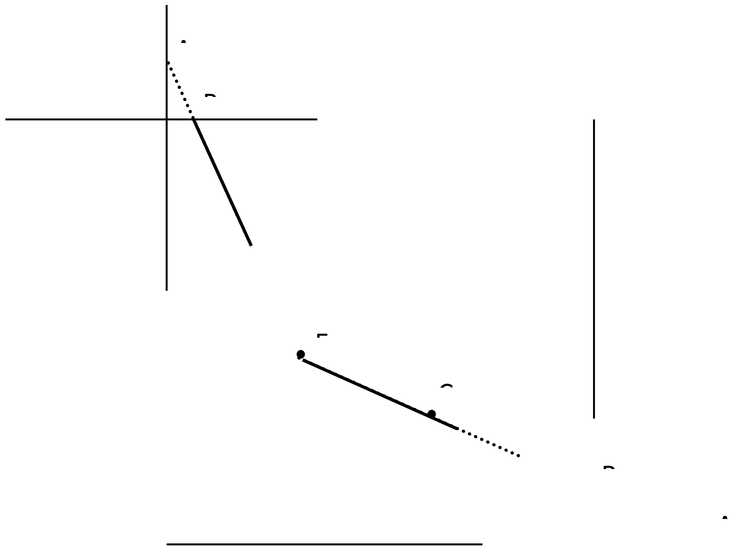
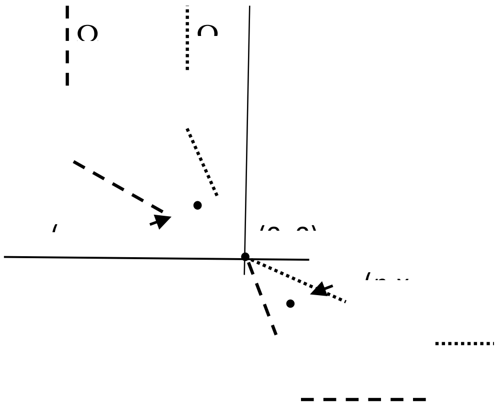


Figure 4: Non-allowable, non-dominated behaviours in the Figure 3 regime



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Appendix: Proofs

A1 The equivalence of two statements of the Opportunity Criterion

In their statement of the Opportunity Criterion, McQuillin and Sugden (2012) use the following definition: for any regime O, $A^*(O)$ is the set of behaviour profiles that are feasible, allowable in O, and for every individual i, not dominated in O for i. Notice that, by definition, $A^*(O) \subseteq A(O)$. Then the criterion is stated as:

> *Opportunity Criterion (as stated by McQuillin and Sugden)*: $\Delta(O)$ is feasible; and for every regime O', $[A(O') \supset A(O)] \Rightarrow [A^*(O') \subseteq A(O)]$.[2]

I now show that this statement is equivalent to the statement in Section A1 of the Appendix, that is:

> *Opportunity Criterion (as stated in Section1of main paper)*: $\Delta(O)$ is feasible; and for every feasible behaviour profile $\Delta' \notin A(O)$, there is some individual i such that Δ'_i is dominated in O_i.

First, suppose that some regime O, for which $\Delta(O)$ is feasible, does not satisfy the McQuillin–Sugden statement of the Opportunity Criterion. Then there is some regime O' such that the following two conditions are satisfied: (i) $A(O') \supset A(O)$, and (ii) there is some feasible behaviour profile Δ' such that $\Delta' \notin A(O)$, $\Delta' \in A(O')$, and for every individual i, Δ' is non-dominated for i in O'. But since (i) implies $O_i \subseteq O'_i$, Δ' can be non-dominated in O' for every i only if it is also non-dominated in O for every i. Thus, the Section 1 statement of the Opportunity Criterion is not satisfied.

Now, suppose that some regime O, for which $\Delta(O)$ is feasible, does not satisfy the Section 1 statement of the Opportunity Criterion. Then there is some feasible behaviour profile $\Delta' \notin A(O)$ such that, for each individual i, Δ'_i is non-dominated in O_i. Now consider the regime O' defined so that, for each individual i, $O'_i = O_i \cup \{\Delta'_i\}$. Notice that this implies $A(O') \supset A(O)$ and $\Delta' \in A(O')$. Since a behaviour cannot dominate itself, Δ'_i can be non-dominated in O_i only if it is also non-dominated in O'_i. Thus, for each individual i, Δ' is non-dominated for i in O'. Thus $\Delta' \in A^*(O')$, which implies *not*$[A^*(O') \subseteq A(O)]$, contrary to the McQuillin–Sugden statement of the Opportunity Criterion. □

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[2] The notation $\supset$ is used to denote 'is a *strict* superset of'.

A2 Proof of the Market Opportunity Theorem

Consider any exchange economy and any market-clearing single-price regime *O*. Let $p \equiv (p_1, ..., p_g)$ be the vector of market prices in this regime. Since *O* is market-clearing, $\Delta(O)$ is feasible. Let Δ' be any feasible behaviour profile. Since Δ' is feasible, net acquisitions of each good *g* by all individuals must sum to zero. So, when valued at the market prices *p*, the total value of net acquisitions for all individuals must sum to zero:

$$\Sigma_i \, \Sigma_{\mathrm{g}} \, p_g \, \Delta'_{i,g} = 0. \tag{A1}$$

Given (A1), one of the following two cases must hold. In *Case 1*, for each individual *i* separately, the total value of net acquisitions is zero. Since *O* is a single-price regime, this implies that for each *i*, Δ'_i must be either allowable or dominated in O_i; the Opportunity Criterion is therefore satisfied. In *Case 2*, there is some individual *j* for whom the total value of net acquisitions is strictly negative. Since *O* is a single-price regime, this implies that Δ'_j must be dominated in O_j; the Opportunity Criterion is therefore satisfied. □

A3 Proof of the Linkage Result

Consider any exchange economy and any regime *O* of that economy that satisfies the Opportunity Criterion. To initiate a proof by contradiction, assume Existence of Preferences, and suppose that $\Delta(O)$ is *not* Pareto-efficient. Let Δ denote the chosen behaviour profile $\Delta(O)$. Then, by the definition of Pareto-efficiency, there is some feasible behaviour profile $\Delta' \neq \Delta$ such that Δ'_i is strictly Pareto-preferred to Δ_i, i.e. such that $\Delta'_i \geq_i \Delta_i$ for every individual *i* and, for some individual *j*, $\Delta'_j >_j \Delta_j$. By the definition of the Opportunity Criterion, *either* (i) Δ' is allowable in *O or* (ii) there is some individual *k* such that Δ'_k is dominated in O_k. First, suppose (i). Then, since Δ'_j is allowable in O_j and $\Delta'_j >_j \Delta_j$, the supposition that *j* chooses Δ_j is inconsistent with Existence of Preferences. So (ii) must be true. Since Δ'_k is dominated in O_k, there must be some Δ''_k that is allowable in O_k, such that Δ''_k dominates Δ'_k. Hence, by Existence of Preferences, $\Delta''_k >_k \Delta'_k$. Since *k* chooses Δ_k when Δ''_k is allowable, Existence of Preferences also implies $\Delta_k \geq \Delta''_k$. So, by transitivity, $\Delta_k > \Delta'_k$. This is inconsistent with the supposition that $\Delta'$is strictly Pareto-preferred to Δ. □

Strong Opportunity Criterion. A regime O satisfies the Strong Opportunity Criterion if $\Delta(O)$ is feasible, and if, for every non-empty set of individuals $S \subseteq I$ and for every behaviour profile Δ'_S that is feasible for S, *either* $\Delta'_S \in A_S(O)$ *or* there is some individual $i \in S$ such that Δ'_i is dominated in O_i.

A4 Proof of the Strong Market Opportunity Theorem

Consider any exchange economy and any market-clearing single-price regime O. Let $p \equiv (p_1, ..., p_g)$ be the vector of market prices in this regime. Since O is market-clearing, $\Delta(O)$ is feasible. Consider any non-empty set of individuals $S \subseteq I$ and any behaviour profile Δ'_S for S that is feasible for S. It is sufficient to prove that *either* (i) for every $i \in S$, Δ'_i is allowable in O_i *or* (ii) there is some individual $j \in S$ such that Δ'_j is dominated in O_j.

Since Δ'_S is feasible for S, net acquisitions of each good g by all individuals in S must sum to zero. So, when valued at the market prices p, the total value of net acquisitions for all individuals in S must sum to zero:

$$\Sigma_{i \in S}\ \Sigma_{\mathrm{g}}\, p_g\, \Delta'_{i,g} = 0. \tag{A2}$$

Given (A2), one of the following two cases must hold. In *Case 1*, for each individual in S separately, the total value of net acquisitions is zero. Since O is a single-price regime, this implies that for each $i \in S$, Δ'_i must be either allowable or dominated in O_i; thus, either (i) or (ii) holds. In *Case 2*, there is some individual $j \in S$ for whom the total value of net acquisitions is strictly negative. Since O is a single-price regime, this implies that Δ'_j must be dominated in O_j; thus, (ii) is satisfied. □

A5 Proof of the Shrinkage Theorem

Let (X^1, O^1) be any pair of an exchange economy and a regime for that economy. For any integer $r > 0$, let (X^r, O^r) be the pair created by combining (X^1, O^1) with r–1 replicas. Suppose that O^r does *not* satisfy the Strong Opportunity Criterion. Then, by definition, there is some non-empty set of individuals $S \subseteq \{1, \ldots, 2r\}$ and some behaviour profile Δ'_S that is feasible for S, such that (i) Δ'_S is not allowable in O^r_S and (ii) for every individual $i \in S$, Δ'_i is not dominated in O^r_i. Now consider (X^{r+1}, O^{r+1}), created by adding a further replica. Since this addition does not affect the endowments or opportunity sets of the members of S, what has been said about Δ'_S must remain true, and so O^{r+1} cannot satisfy the Strong Opportunity Criterion. □

A6 Proof of the Convergence Theorem

Let (X^1, O^1) be any pair of an exchange economy and a regime for that economy. For each individual *i*, let Ω_i be the set of behaviours for i that are neither allowable nor dominated in O^1_i, defined in a space of net acquisitions that is common to all individuals. Let Ω be the convex hull of the sets Ω_i for $i = 1, \ldots, N$.

Take any $\varepsilon > 0$. Let $\Omega^*_i(\varepsilon)$ be the set of behaviours for *i* that are neither ε-allowable nor ε-dominated in O^1_i. Let $\Omega^*(\varepsilon)$ be the convex hull of the sets $\Omega^*_i(\varepsilon)$ for $i = 1, \ldots, N$. By the definitions of ε-allowability and ε-dominance, for each *i*, every element of $\Omega^*_i(\varepsilon)$ is strictly in the interior of Ω_i. Consider any behaviour $b^* \in \Omega^*(\varepsilon)$. By the definition of a convex hull, b^* can be constructed by mixing the elements of some set of behaviours $\{b_1,\ldots, b_m\}$, where each b_j is an element of $\Omega^*_i(\varepsilon)$ for some *i*; I will say that *i* is the *actor* for b_j. Each b_j has a real-valued weight α_j in this mixture, where $0 < \alpha_j \leq 1$ and $\Sigma_j\, \alpha_j = 1$. These weights need not be rational numbers. However, because the rational numbers form a dense subset of the real numbers, we can construct behaviours $b'_1, \ldots, b'_m$, such that each b'_j is sufficiently close to the corresponding b_j that it is an element of Ω_i for the individual *i* who is the actor for b_j, and b^* is a convex combination of $b'_1, \ldots, b'_m$ in which each of the weights $\alpha'_1, \ldots, \alpha'_m$ is a strictly positive *rational* number. (To ensure that non-negativity constraints are not violated, each b'_j can be required to be a convex combination of b_j and b^*.) Now consider the economy and regime (X^r, O^r) created by combining (X^1, O^1) with r–1 replicas, for some $r \geq 1$. Because of the results established in the previous paragraph, if *r* is sufficiently large, we can construct a non-empty set $S \subseteq I$ of individuals, and a behaviour profile Δ_S for *S*, with the following properties. First, *S* can be partitioned into non-empty subsets $S_1, \ldots, S_m$, such that for each $j = 1, \ldots, m$, the ratio between the number of individuals in S_j and the number of individuals in *S* is α'_j. Second, for each $j = 1, \ldots, m$, each individual in S_j is a replica of the individual who is the actor for b'_j. Third, for each $j = 1, \ldots, m$, the behaviour Δ_i for each individual *i* in S_j is b'_j. Given these properties, Δ_S is feasible for *S* if and only if $b^* = \mathbf{0}$.

I now show that if the Strong Opportunity Criterion is satisfied for all $r > 0$, $\mathbf{0} \notin \Omega^*(\varepsilon)$. To initiate a proof by contradiction, suppose that $\mathbf{0} \in \Omega^*(\varepsilon)$, and set $b^* = \mathbf{0}$. Then if *r* is sufficiently large, there exists a non-empty set $S \subseteq I$ of individuals, and a feasible behaviour profile Δ_S for S, such that, for each $i \in S$, $\Delta_i \in \Omega_i$ (i.e. Δ_i is neither allowable nor dominated in $O^r{}_i$). This implies that O^r does not satisfy the Strong Opportunity Criterion. Thus, if the Strong Opportunity Criterion is satisfied for all $r > 0$, $\mathbf{0} \notin \Omega^*(\varepsilon)$.

Now suppose that the Strong Opportunity Criterion is satisfied for all $r > 0$. Since $\Omega^*(\varepsilon)$ is a convex set by construction, $\mathbf{0} \notin \Omega^*(\varepsilon)$ implies that there is some hyperplane through $\mathbf{0}$ that does not intersect $\Omega^*(\varepsilon)$, and hence does not intersect any $\Omega^*{}_i(\varepsilon)$. Thus, by the definition of $\Omega_i^*(\varepsilon)$, for each individual $i = 1, \ldots, rN$, every behaviour on this hyperplane is either ε-allowable or ε-dominated in $O^r{}_i$. Equivalently, there exists a finite, real-valued price vector $p = (p_1, \ldots, p_G)$ such that, for each individual i, every behaviour Δ_i that satisfies $\Sigma_g\, p_g\, \Delta_{i,g} = 0$ is either ε-allowable or ε-dominated in $O^r{}_i$, i.e., O^r is an ε-single-price regime. □

A7 Proof of Existence Theorem

Consider any exchange economy. Assume that Continuity and Intrinsic Value of Money are satisfied. Let P be the set of price vectors p that satisfy the condition that, for each non-money good $g = 2, \ldots, G$, $p_g^{\mathrm{H}} \geq p_g \geq p_g^{\mathrm{L}}$. For any such price vector p, for each good g = 1, …, G, let $x_g(p)$ be the net excess demand for good g that would occur if p was the price vector in a strict single-price regime. Adapting a concept from Walrasian general equilibrium theory, I define a *tâtonnement function* $f: P \rightarrow P$. For the purposes of the proof, this is a mathematical construction and nothing more. However, it may be helpful to think of the function as a model of how, in a market economy, prices might adjust in response to excess demands and excess supplies.

As a first step, I define $\phi(z) = \alpha(1 - e^{-z})/(1 + e^{-z})$ for all real numbers z, where e is Euler's number and α is some constant satisfying $1 \geq \alpha > 0$. Notice that $\phi(.)$ is a continuous and monotonically increasing function with $\phi(0) = 0$; $\phi(z) \rightarrow -\alpha$ as $z \rightarrow -\infty$, and $\phi(z) \rightarrow \alpha$ as $z \rightarrow \infty$. Writing $f(p)$ as $[f_1(p), \ldots, f_G(p)]$, I define $f_g(p)$ for $g = 2, \ldots, G$ by:

(1a) $f_g(p) = (1 - \phi[x_g(p)])p_g + \phi[x_g(p)]p_g^{\mathrm{H}}$ *if* $x_g(p) \geq 0$; *and*

(1b) $f_g(p) = (1 - \phi[x_g(p)])p_g + \phi[x_g(p)]p_g^{\mathrm{L}}$ *if* $x_g(p) \leq 0$.

Because of Walras's Law, these equations also define $f_1(p)$. Because $\phi(.)$ is a continuous function, and because (by Continuity) each $x_g(.)$ is a continuous function, $f(.)$ is a continuous function from P to P. By construction, P is a closed, bounded, convex set. Thus, by Brouwer's Theorem, there is a fixed point $p^* \in P$ such that $f(p^*) = p^*$.

Consider any such p^*. First, suppose there is some non-money good g such that *either* $p^*_g = p_g^{\mathrm{H}}$ *or* $p^*_g = p_g^{\mathrm{L}}$. So, by Intrinsic Value of Money, $x_1(p^*) > 0$. ByWalras's Law, there must be some non-money good h (which may or may not be g) for which the value of net excess demand is strictly negative, i.e. $p^*_h\, x_h(p^*) < 0$. By the definition of p^*, $f_h(p^*) = p^*_h$. Thus, *either* $p^*_h > 0$ and $x_h(p^*) < 0$ (Case 1), *or* $p^*_h < 0$ and $x_h(p^*) > 0$ (Case 2). Suppose Case 1 holds. By (1b), $[f_h(p^*) = p^*_h$ and $x_h(p^*) < 0]$ implies $p^*_h = p_h^{\mathrm{L}} < 0$, a contradiction. Suppose Case 2 holds. By (1a), $[f_h(p^*) = p^*_h$ and $x_h(p^*) > 0]$ implies $p^*_h = p_h^{\mathrm{H}} > 0$, a contradiction. So the original supposition is false. That is, for every non-money good g, $p_g^{\mathrm{H}} > p^*_g > p_g^{\mathrm{L}}$.

It then follows from (1a) and (1b) that, for each non-money good g, $f_g(p^*) = p^*_g$ implies $x_g(p^*) = 0$. Thus, there exists a single-price market-clearing regime, namely the strict single-price regime in which the price vector is p^*. □